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The General Three-Vertex, BRS Identities and Renormalizability of the Light-Cone Gauge

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Abstract. The general one-loop three-vertex $\Gamma^{abc}_{\mu\nu\lambda}(p,q,r)$ in the four-component formulation of the Yang-Mills theory is calculated in the light-cone gauge. The nonvanishing counter Lagrangian constructed from this three-vertex and the self-energy is proportional to the original Lagrangian, the single renormalization constant being $-11g^2 C_{\mathrm{YM}}\Gamma(2-\omega)/48\pi^2$. Gauge dependent and nonlocal counterterms do not contribute to the renormalization constant, but are needed to verify the appropriate Slavnov-Taylor (ST) and Becchi-Rouet-Stora (BRS) identities.

1. Introduction

The light-cone gauge

$$\underline{A}^+ \equiv n^+_\mu \underline{A}^\mu = 0, \qquad (n^+)^2 = 0 \tag{1.1}$$

has recently become very popular in the study of quantum field theories [1]. The gauge is not only ghost-free but can also be used to remove from the theory all unphysical degrees of freedom associated with gauge transformations [2]. For example, the $SO(1,3)$ Lorentz symmetry among the four components of the gauge field in the Yang-Mills theory is reduced in the light-cone gauge to a $SO(2)$ symmetry of the two physical components. The gauge has been used to prove the finiteness of the $N=4$ supersymmetric theory [3]; it renders the study of the quantum dynamic Kaluza-Klein theories especially simple [4]; it is also the only gauge in which a quantum formulation of superstring theories is known [5].

Although the light-cone gauge has been known for some time, the technique for computing Feynman integrals in the gauge was mastered [6] only after the discovery not long ago of the correct prescription [7] for the operator $1/p^+$. Recently, based on calculations of the self-energy and certain special three- and four-point functions at the one-loop order, the one-loop renormalizability of the physical sector of the Yang-Mills theory in the light-cone gauge was demonstrated [8].

The light-cone gauge permits two formulations [8]: a four-component formulation with the $SO(1,3)$ symmetry (LCM4) and a two-compoent one with $SO(2)$ symmetry (LCM2, in which the two unphysical degrees of freedom are eliminated using the gauge constraint (1.1) and an equation of motion (6.3) conjugate to it). Only LCM4 retains the gauge degree of freedom which causes it to be more cumbersome than LCM2 in the computation of gauge independent quantities. On the other hand, LCM4 admits Slavnov-Taylor [9] and the usual Becchi-Rouet-Stora identities [10]. One of the issues not settled in previous studies is whether all the infinite parts of radiatively corrected vertex functions in LCM4, even those that do not contribute to the renormalization constant, are needed to satisfy the appropriate ST and BRS identities [11]. This paper is addressed to that issue.

We have calculated the general one-loop three-vertex (p,q,r) in LCM4. Our result agrees with that recently reported by Dalbosco [12]. The result confirms earlier conclusions [8] regarding the one-loop renormalizability of LCM4, in particular that it entails a single renormalization constant identical to that computed for LCM2. However, the three-vertex generates many counterterms, some of which are nonlocal, that do not contribute to the renormalization constant. We show that the presence of such terms is actually needed to satisfy the ST and BRS identities, which are shown to be equivalent.

In Sect. 2 we construct all the allowed rank-3 tensors of which the three-vertex can be composed. In Sect. 3 we briefly describe the calculation of the infinite parts of the one-loop three-vertex and give the result, showing that it involves most, but not all of the allowed tensors. In Sect. 4 it is shown that the complete three-vertex is needed to satisfy the three-point ST identity. Two special limits of this identity are also examined. In Sect. 5 it is shown that, in the limit corresponding to (1.1), BRS identities have exact ST counterparts and are satisfied. In Sect. 6 we demonstrate the one-loop renormalizability of LCM4. A summary and concluding remarks are given in Sect. 7.

2. Allowed Rank-3 Tensors

Here we enumerate all rank-3 tensors with which the three-vertex can be constructed. First define the reduced three-vertex by factorizing out the structure constant f^{abc},

$$\Gamma^{a_1 a_2 a_3}_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) \equiv f^{a_1 a_2 a_3} \tilde{\Gamma}_{m_1 m_2 m_3} \tag{2.1}$$

where for convenience a middle Latin letter has been used to represent a Lorentz index and a momentum

$$m_i = (\mu_i, p_i). \tag{2.2}$$

Because Γ is symmetric under any permutation of the labels 1, 2 and 3 while f is antisymmetric, $\tilde{\Gamma}$ must also be antisymmetric and satisfy the properties

$$\tilde{\Gamma}_{mnl} = -\tilde{\Gamma}_{mln} = -\tilde{\Gamma}_{nml} = -\tilde{\Gamma}_{lnm}. \tag{2.3}$$

Let $T^{(i)}_{mnl}$ be a set of tensors each of which is antisymmetric under the exchange

$$n \leftrightarrow l$$

but has no specific symmetry under

$$m \leftrightarrow n \quad \text{and/or} \quad m \leftrightarrow l$$

then $\tilde{\Gamma}$ can be expanded in terms of T

$$\tilde{\Gamma}_{mnl} = \sum_i a_i [T^{(i)}_{mnl} + T^{(i)}_{nlm} + T^{(i)}_{lmn}] \tag{2.4}$$

where a_i are constant coefficients.

The tensor functions $T^{(i)}$ can be constructed from the tensor $g_{\mu\nu}$, the vectors p, q, r, n^+ and n^-, the components $p^+, p^-, \ldots$ and the scalars p^2, $p \cdot q$, and so on, subject to the following constraints:

(i) If one thinks of the superscripts "+" and "−" as representing "charges", then in any tensor expansion the total charge conserved. This is a consequence of the conservation of magnetic quantum number associated with $SO(3,1)$.

(ii) (Let t represent any of the external momenta p, q and r so that by t^2 we mean p^2, or $p \cdot q$ or $q \cdot r$ and so on and by $t^+ t^-$ we mean $p^+ p^-$ or $r^+ p^-$ and so on.) In an allowed tensor, a factor t^- in the denominator is always accompanied by another factor t^+ in the denominator. This rule follows from the fact that factors of k^-, where k is the integration variable, occurs only in the numerator of light-cone gauge integrands, for which the Mandelstam-Leibbrandt prescription [7] yields the result [8]

$$\begin{aligned} &\int d^{2\omega} k (k^-)^n (k^+)^m \text{ (neutrally charged integrand)} \\ &= (t^-)^n (t^+)^m \text{ (neutral power series of } t^2/t^+ t^-). \end{aligned}$$

A consequence of this rule is that a tensor such as

$$\frac{p^2}{q^- r^-} p_\mu n^-_\nu n^-_\lambda$$

is not allowed.

(iii) The tensor must have the dimension of momentum.

(iv) The tensor must be regular when any one (or more) of the three momenta p, q and r, or any one of their squares vanishes. Thus the tensor

$$\frac{p^+(q-r)^+}{p^2} p_\mu n^-_\nu n^-_\lambda$$

is not allowed because it is irregular when p^2 is zero. On the other hand

$$\frac{p^-}{p^+} (q-r)_\mu n^+_\nu n^+_\lambda$$

is allowed because the Mandelstam-Leibbrandt prescription [7] for the operator $1/p^+$ is such that

$$\lim_{p \to 0} 1/p^+ = 0.$$

In constructing the tensor functions, it is convenient to factorize the tensor as

$$T^{(i,+)}_{\mu\nu\lambda} A_i(p,q,r) \quad \text{or} \quad T^{(i,-)}_{\mu\nu\lambda} S_i(p,q,r)$$

where $T^{(i,+)}_{\mu\nu\lambda} (T^{(i,-)}_{\mu\nu\lambda})$ is a tensor constructed from the vectors p, q, r, n^+ and n^- and $g_{\mu\nu}$ which is sym-