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Anticentrifugal Stretching in ^{20}Ne

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A variation-after-projection Hartree-Fock calculation shows that the rms radius of ^{20}Ne decreases with increasing spin in the ground-state band. This decrease is consistent with the significant reduction in α width between the 6^+ and the 8^+ states. It also leads to the theoretical $B(E2)$ values reproducing the measured values exceedingly well.

The atomic nucleus ^{20}Ne has provided one of the classic manifestations of collective rotational motion in light nuclei.[1] The low-lying members of its ground-state band have energy spacings proportional to $J(J+1)$ and have the strong intraband $E2$ transitions characteristic of the rigid rotor. However, higher-lying members of the band, especially the recently observed 8^+ state at 11.95 MeV,[2,3] have properties which are markedly different from those of the simple rotational model. Thus in the rigid rotor the 8^+ state is predicted to be at a considerably higher energy than 12 MeV, and the ratio of the $E2$ γ-decay strengths, $\mathcal{R} = B(E2; 6^+ \to 4^+)/B(E2; 8^+ \to 6^+)$, which is predicted to be 0.96, is measured[3] to be $2.7^{+1.9}_{-0.9}$. On the other hand both the j-j coupled[4] and the SU(3)[5] shell-model calculations predict $\mathcal{R} = 1.6$ and level spacings close to those observed. In these models an (additional) effective charge of $\sim 0.5e$ per nucleon is needed to reproduce the absolute $B(E2)$ values.

The α decay of excited states is another interesting facet of the structure of ^{20}Ne. Among members of the ground band the 6^+ and 8^+ states are α-particle unbound, and the α widths of these states have recently been measured,[2] the result being $\Gamma_{6^+}{}^{\alpha} = 110 \pm 25$ eV and $\Gamma_{8^+}{}^{\alpha} = 35 \pm 10$ eV. As usual one analyzes the experimental width Γ_l in terms of the product of a spectroscopic factor S_l and a single-particle width $\Gamma_l{}^{\text{s.p.}}$, or $\Gamma_l{}^{\text{exp}} = S_l \Gamma_l{}^{\text{s.p.}}$. Here the "single particle" has reduced mass number $\frac{16}{5}$, and its motion is the relative motion of the departing α particle and the residual ^{16}O. Arima and Yoshida[6] calculated $\Gamma_l{}^{\text{s.p.}}$ using the Coulomb potential plus a real Woods-Saxon potential of appropriate depth, radius R, and diffusivity a; these parameters are chosen in such a way that the wave function has maximum overlap with the wave function obtained in the cluster model[6] and that the resonance energy coincides with the observed energy. The following results[6] were then obtained: (a) Assuming $R_{6^+} = R_{8^+}$, then one must have $S_{6^+} = 2S_{8^+} = 0.24$ in order to reproduce the experimental α widths; (b) if one demands that $S_{6^+} = S_{8^+} = 0.24$ then $R_{6^+} - R_{8^+} \cong 0.25$ F. Neither (a) nor (b) is compatible with the shell models mentioned previously. However, since $S_{6^+}=S_{8^+}=0.24$ is predicted in their α-cluster model, Arima and Yoshida favor the second possibility and interpret the α-decay data as experimental evidence of anticentrifugal stretching in ^{20}Ne.

In this Letter we report results obtained in many-body variational calculations for ^{20}Ne. These results reproduce, without the help of an effective charge, all absolute measured $B(E2)$ values in the ground band and confirm the speculation of Arima and Yoshida that $R_{8^+}<R_{6^+}$. As we shall see, the two phenomena are actually intimately related. This then presents a unified picture for the ^{20}Ne ground band. Our variational calculations are carried out with the projected Hartree-Fock (PHF) method, where the projection of angular momentum is done *before* variation. We use a five major-shell oscillator basis to expand the single-particle wave functions. The frequency of the oscillator is determined by the variational principle.[7] The Hamiltonian is $H=t_{\text{rel}}+v_N+v_{\text{em}}$, where t_{rel} is the total kinetic energy *with respect to* the center of mass, v_N is the two-body N-N interaction, here represented by the Saunier-Pearson potential No. 2,[8] and v_{em} is the two-body Coulomb interaction plus the one-body approximated electromagnetic spin-orbit interaction.[9] We define the intrinsic Hamiltonian as

$$H_{\text{int}}(\lambda)\equiv H_{\text{int}}(\lambda_J,\lambda_Q)=H-\lambda_J J^2-\lambda_Q Q_2^{\,0}, \tag{1}$$

where $Q_2^{\,0}$ is the mass quadrupole operator. For given values of λ's, the single-particle wave functions are determined by the Hartree-Fock equation

$$\delta\langle\psi(\lambda)\,|\,H_{\text{int}}(\lambda)\,|\,\psi(\lambda)\rangle=0. \tag{2}$$

For each state of good J the variational parameters are determined such that

$$\frac{\partial E_J}{\partial\lambda}=\frac{\partial}{\partial\lambda}\left[\frac{\langle\psi(\lambda)\,|H\hat{P}_J|\psi(\lambda)\rangle}{\langle\psi(\lambda)\,|\hat{P}_J\,|\psi(\lambda)\rangle}\right]=0, \tag{3}$$

where $\hat{P}_J$ is the angular momentum projection operator. In practice since the optimum value for λ_J is very similar throughout the band, the average value of $\lambda_J=0.27$ is taken, and only λ_Q is varied in the final computation. Technical details of the calculations are found elsewhere.[7,10]

In Fig. 1 the energies of the J-projected states in the ground band of ^{20}Ne are plotted versus the variational parameter $\lambda=\lambda_Q$, as well as the suitably defined[7] quadrupole deformation parameter β. One observes that as the spin increases, the optimum value of λ or β decreases. The trend

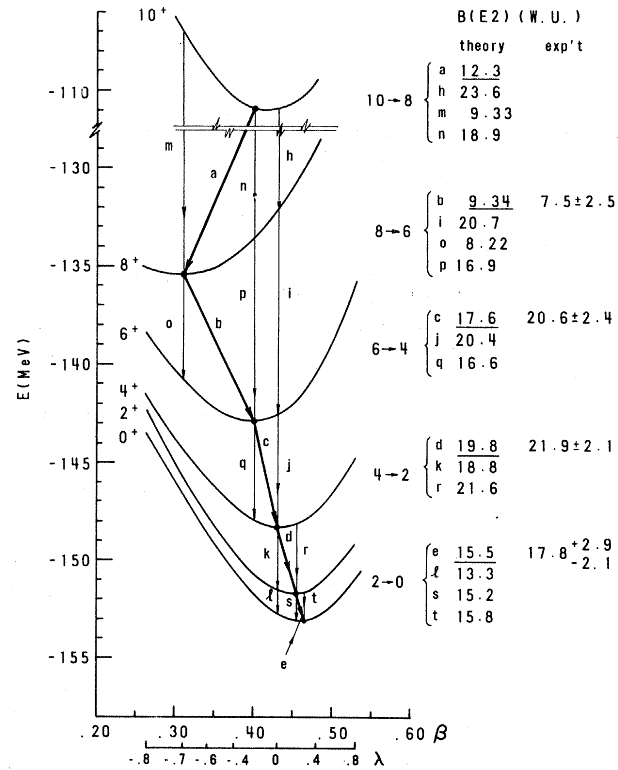

FIG. 1. Projected energies of the ground-band states in ^{20}Ne versus the mass quadrupole cranking parameter λ as defined in the constrained Hamiltonian $H_I=H-0.27J^2-\lambda Q_2^{\,0}$. The abscissa is linear in β, the quadrupole deformation parameter as defined in Ref. 7. $B(E2)$ values for the transitions indicated by lettered arrows are also given in Weisskopf units (1 W.u. $=3.23e^2$ F^4). The transitions a, b, c, d, and e occur between states at the energy minima.

is broken only for the 10^+ state, which has components mainly outside the s-d shell. A very similar effect is also observed in the α-cluster calculation for ^{20}Ne by Horiuchi,[11] where the distance between the centers of mass of the ^{16}O and α clusters is taken as a variational parameter. Various computed $B(E2)$ strengths are also shown in Fig. 1. The $B(E2)$ values of the transitions a–e (technically these are "cross-band" transitions) between the optimum states agrees exceedingly well with the measured[3] values. A value of 1.9 for $\mathcal{R}$ is predicted. In contrast, for the "in-band" transitions, particularly the sequence h–l, which is obtained with $\lambda_Q=0$, the rotational values are predicted, and $\mathcal{R}=0.98$.

Because the 8^+ state (hereafter a state is understood to be that obtained at the optimum value of λ_Q for that spin) is less deformed than the 0^+

TABLE I. Change in ρ, the $\alpha - {}^{16}\mathrm{O}$ cluster distance in the $^{20}\mathrm{Ne}$ ground-state band.

J^+	R_{rms} (^{20}Ne) (F)	$\rho(\alpha - {}^{16}\mathrm{O})$ $R_{16}=2.35$ (F)	$R_{16}=2.50$ (F)	$\Delta\rho = \rho_J - \rho_0$ $R_{16}=2.35$ (F)	$R_{16}=2.50$ (F)
0^+	2.678	3.682	3.312	0	0
2^+	2.673	3.840	3.287	0.022	0.025
4^+	2.655	3.761	3.194	0.101	0.118
6^+	2.635	3.672	3.089	0.190	0.223
8^+	2.606	3.540	2.931	0.322	0.381
10^+	2.674	3.844	3.292	0.018	0.020

state, one expects the former to have a smaller radius. In column 2 of Table I the rms radii for states in the ground band, calculated *with respect to* the center of mass, are listed. An anticentrifugal-stretching effect is evident, as from 0^+ to 8^+ the rms radius decreases by ~3%. This decrease can be related to the change of ρ, the rms distance between the centers of mass of ^{16}O and α in a clustering state of ^{20}Ne, through the equation

$$20R_{^{20}\mathrm{Ne}}{}^2 = 16_{^{16}\mathrm{O}}{}^2 + 4R_\alpha{}^2 + \tfrac{16}{5}\rho^2. \qquad (4)$$

Table I shows ρ and $\Delta\rho_J = \rho_{J^+} - \rho_{0^+}$, for $R_\alpha = 1.44$ F and two fixed values for $R_{^{16}\mathrm{O}}$. We see that $\rho_{6^+} - \rho_{8^+} = 0.13$ to 0.16 F, which agrees reasonably well with the change of 0.25 F in the radii of the Woods-Saxon potentials of Arima and Yoshida.[6] A more direct comparison would be made if we could compare ρ with the rms radii of the internally normalized resonance wave functions obtained by these authors. Though these radii are not available, the size parameters were given[6] for the harmonic oscillator wave functions having maximum overlap with the resonance wave functions in the internal region. Taking $\nu = 0.740$ F^{-2} for the 6^+ state and $\nu = 0.851$ F^{-2} for the 8^+ state, and knowing that the oscillator wave functions have eight quanta, we find $\rho_{6^+} = 3.58$ F and $\rho_{8^+} = 3.34$ F. This represents an isomer shift of 0.24 F. In other words, the isomer shift in the radius of the Woods-Saxon well is essentially equal to the isomer shift in ρ. If we now take into consideration the experimental errors[2] in the α widths, then from Table 2 of Ref. 6 the radii of the Woods-Saxon wells for 6^+ and 8^+ are $3.81^{+0.09}_{-0.12}$ and $3.52^{+0.09}_{-0.11}$ F, respectively. From the analysis given above this implies $\rho_{6^+} \simeq 3.61 \pm 0.10$ F and $\rho_{8^+} \simeq 3.33 \pm 0.10$ F, or an isomer shift of 0.28 ± 0.20 F, which, considering the size of the error, is in reasonable agreement with our prediction.

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