1.2×10^{-4}.

A branching-ratio limit of this size does not put very restrictive limits on the matrix elements of an isospin-breaking term in the Hamiltonian. The smaller energy available to the excited-state β decay already inhibits the branch by the ratio of the Fermi functions; so the experimental 50% confidence limit on the branching ratio implies a limit on the matrix element of

$$\left|\frac{M(0_1^+\to0_2^+)}{M(0_1^+\to0_1^+)}\right|=\left\{R\frac{f(0_1^+\to0_1^+)}{f(0_1^+\to0_2^+)}\right\}^{1/2}<0.029. \quad (3)$$

The experiment requires, then, that the difference in the dynamic-distortion matrix elements in the two nuclei is less than about 50 keV [see Eq. (2)].

To see whether this is reasonable, it is necessary to specify the isospin-conserving wave functions φ_k. These states are consistent with two-particle and four-particle, two-hole configurations built on a ^{40}Ca core.[4] Using the wave functions determined in Ref. 4, one can calculate the Coulomb mixing of the two states. Rappleyea and Kunz[5] have carried out an analogous calculation; they have recomputed wave functions in the manner of Ref. 4, including the Coulomb interaction, obtaining the wave functions Ψ_k directly. Their predicted value for the matrix-element ratio is

$$|M(0_1^+\to0_2^+)/M(0_1^+\to0_1^+)|=0.020, \quad (4)$$

which is consistent with the results of this experiment.

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Ground-State Correlation and the Isobaric Analog State

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For an $N\neq Z$ system interacting through a Hamiltonian including the Coulomb interaction, the effect of proton-neutron correlations on the ground state and its isobaric analog is discussed. It is shown that the correlation greatly reduces the isospin impurity in the ground state.

It is well known that for $N\neq Z$ systems isospin is not a good quantum number for the uncorrelated ground-state wave function $|0\rangle$ obtained in the Hartree-Fock (HF) approximation, even when the isospin-symmetry-breaking Coulomb interaction is *not* included in the Hamiltonian. This in fact is a special case of a general property of the variational method utilized in the HF approximation, that $|0\rangle$ is not necessarily an eigenfunction of any operator which commutes with the Hamiltonian. Recently Engelbrecht and Lemmer[1] showed that, in the absence of the Coulomb interaction, the isospin symmetry of $|0\rangle$ is restored if proton-neutron correlations, generated within the framework of the random phase approximation (RPA), are incorporated into it. In this Letter we first explore their notion in more detail, and then introduce the Coulomb interaction into the picture. It is shown that when all the single-particle (sp) Coulomb energies of the protons are degenerate, the correlated ground state $|\tilde{0}\rangle$ is still isospin pure, and the Coulomb displacement energy of its isobaric analog [i.e., the isobaric analog state (IAS)] is equal to the degenerate sp Coulomb energy. This simple picture is disdisturbed, however, when the sp Coulomb energies are nondegenerate. We calculate the shift of the Coulomb displacement energy of the IAS and the isospin impurity in $|\tilde{0}\rangle$ for a simple but very physical nondegenerate case.

We start by considering proton-particle, neutron-hole excitations created out of $|\tilde{0}\rangle$ by the (approximate) boson operators

$$B_\lambda^\dagger(t) \equiv \exp(-i\Omega_\lambda t)B_\lambda^\dagger \equiv \exp(-i\Omega_\lambda t)(\sum_\alpha x_\alpha A_\alpha^\dagger - \sum_{\bar\alpha} y_{\bar\alpha} A_{\bar\alpha}), \quad (1)$$

where α stands for the particle-hole pair[2] $(a\bar{i})$ and $\bar\alpha$ stands for $(\bar{a}i)$. We use $a, b, \cdots$ $(\bar{a}, \bar{b}, \cdots)$ and $i, j, \cdots$ $(\bar{i}, \bar{j}, \cdots)$, respectively, to label the unoccupied (occupied) proton and neutron orbitals. For future reference we may separate the set $\{\alpha\}$ into the set $\{\alpha_0\} = \{(a_0\bar{i}_0)\}$, where the subscript label zero denotes the orbitals of the excess neutrons and their (unoccupied) proton counterparts, and the remainder set[2] $\{\alpha'\}$. Also, in (1), $A_\alpha^\dagger \equiv p_a^\dagger n_{\bar{i}}$, $A_{\bar\alpha} \equiv n_i^\dagger p_{\bar{a}}$, and $A_\alpha|0\rangle \equiv 0$, $B_\lambda|\tilde{0}\rangle \equiv 0$.

Suppressing the Coulomb interaction for the moment, the linearization of the equations of motion of $B_\lambda^\dagger(t)$ and $B_\lambda(t)$ leads to the RPA-like equations[1]

$$(\epsilon_\alpha - \Omega_\lambda)x_\alpha + \sum_\beta F_{\alpha\beta}x_\beta + \sum_{\bar\beta} F_{\alpha\bar\beta}y_{\bar\beta} = 0, \quad (2a)$$

$$(\epsilon_{\bar\alpha} + \Omega_\lambda)y_{\bar\alpha} + \sum_\beta F_{\bar\alpha\beta}x_\beta + \sum_{\bar\beta} F_{\bar\alpha\bar\beta}y_{\bar\beta} = 0. \quad (2b)$$

In (2), $\epsilon_\alpha = \epsilon_a^{\,p} - \epsilon_{\bar{i}}^{\,n}$ and $\epsilon_{\bar\alpha} = \epsilon_i^{\,n} - \epsilon_{\bar{a}}^{\,p}$, where the ϵ^n (ϵ^p) are HF neutron (proton) sp energies and $F_{\alpha\beta} = F_{\beta\alpha} = \tilde{v}(a_\alpha \bar{i}_\beta \bar{i}_\alpha a_\beta)$ are antisymmetrized, particle-hole matrix elements of the isovector components of the N-N interaction v. $F_{\alpha\bar\beta} = F_{\bar\beta\alpha}$ and $F_{\bar\alpha\bar\beta}$ are similarly defined.

Engelbrecht and Lemmer[1] pointed out that the particle-hole approximated isospin-lowering operator

$$T_- = \sum_\alpha c_\alpha A_\alpha^\dagger + \sum_{\bar\alpha} c_{\bar\alpha} A_{\bar\alpha}, \quad (3)$$

where $c_\alpha = \langle a|\bar{i}\rangle$, $c_{\bar\alpha} = \langle\bar{a}|i\rangle$ are overlaps of the sp proton and neutron wave functions, satisfies the conditions (4) and creates the IAS with excitation energy $\Omega_{\rm IAS} = 0$. This is because the relations

$$\epsilon_\alpha c_\alpha + \sum_\beta F_{\alpha\beta}c_\beta - \sum_{\bar\beta} F_{\alpha\bar\beta}c_{\bar\beta} = 0, \quad (4a)$$

$$-\epsilon_{\bar\alpha}c_{\bar\alpha} + \sum_\beta F_{\bar\alpha\beta}c_\beta - \sum_{\bar\beta} F_{\bar\alpha\bar\beta}c_{\bar\beta} = 0 \quad (4b)$$

are automatically satisfied in the self-consistent HF basis. It then follows that $|\tilde{0}\rangle$ and its IAS, $T_-|\tilde{0}\rangle$, must each have good isospin and all remaining $B_\lambda^\dagger|\tilde{0}\rangle$ states are antianalogs.

We now introduce the Coulomb interaction into the Hamiltonian. It is easy to see that the only effect of this on (2) is to replace ϵ_α by $\epsilon_\alpha + \epsilon_a^{\,c}$ and $\epsilon_{\bar\alpha}$ by $\epsilon_{\bar\alpha} - \epsilon_{\bar\alpha}^{\,c}$; the ϵ^c are the sp Coulomb energies. On the other hand (4) still holds. In the trivial case where all of the ϵ^c are degenerate, $\epsilon^c = \epsilon_0^{\,c}$, T_- again satisfies (2) and all that has been said about $|\tilde{0}\rangle$ and its proton-particle, neutron-hole excitations still holds except that now $\Omega_{\rm IAS} = \epsilon_0^{\,c}$. This is an encouraging result since experimentally[3] $\Omega_{\rm IAS}$ is in fact very close to $\epsilon_0^{\,c}$.

To explore the structure of (2) in more detail as well as to prepare for the case of nondegenerate ϵ^c, we now borrow the schematic model,[4] which assumes that the F matrix in (4) is factorizable, i.e., $F_{\alpha\beta} = \mu F_\alpha F_\beta$, etc. With this assumption, the F matrix can be eliminated from (2) and (4) and we get the dispersion expression

$$g(\Omega) \equiv \sum_\alpha \frac{\epsilon_\alpha^{\,2}c_\alpha^{\,2}}{\epsilon_\alpha + \epsilon_a^{\,c} - \Omega} + \sum_{\bar\alpha} \frac{\epsilon_{\bar\alpha}^{\,2}c_{\bar\alpha}^{\,2}}{\epsilon_{\bar\alpha} - \epsilon_{\bar\alpha}^{\,c} + \Omega} = \sum_{\alpha_0} \epsilon_{\alpha_0}c_{\alpha_0}^{\,2} + \sum_{\alpha} \epsilon_{\alpha'}c_{\alpha'}^{\,2} + \sum_{\bar\alpha} \epsilon_{\bar\alpha}c_{\bar\alpha}^{\,2}. \quad (5)$$

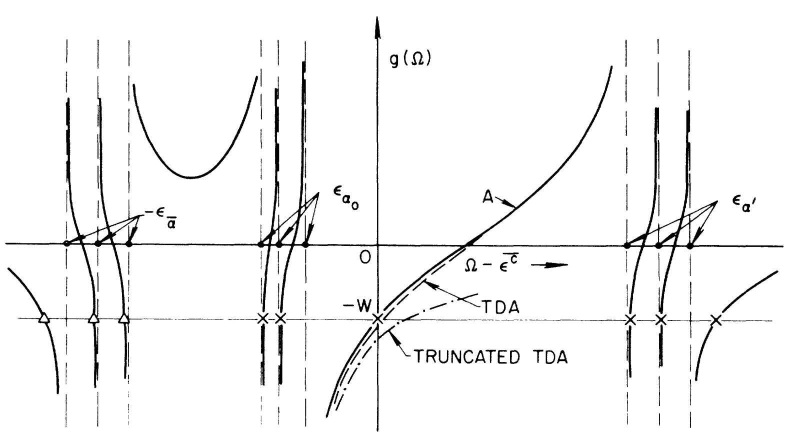

FIG. 1. Solution of the schematic-model dispersion formula (5); $-W < 0$ is equal to the right-hand side of (5). The crosses are for physical solutions and the triangles are for the nonphysical ones. A is the branch of the dispersion curve which leads to the IAS. The corresponding curves in TDA (dashed line) and in truncated TDA (dot-dashed line) are indicated.

It is evidently clear that for degenerate ϵ^c, $\Omega = \epsilon_0^c$ is a solution of (5). For $N > Z$ systems, because of the symmetry energy, $\epsilon_{\alpha_0} = \epsilon_{a_0} - \epsilon_{\bar{i}_0} < 0$. Furthermore, since $|c_{\alpha_0}| \lesssim 1$, and $|c_{\alpha'}| \approx |c_{\bar{\alpha}}| \ll 1$, the right-hand side of (5) is dominated by the first term and must be less than zero. In Fig. 1, $g(\Omega)$ is plotted against $\Omega - \epsilon_0^c$, for the case of degenerate ϵ^c. It is now easy to examine the effect on Ω_{IAS} when different approximations are used. In the Tamm-Dancoff approximation (TDA) the poles at $\Omega - \epsilon_0^c = -\epsilon_{\bar{\alpha}}$ are neglected. From (5) it can be shown that the effect on Ω_{IAS} is that it will be shifted by $\delta\Omega = \sum_{\bar{\alpha}} c_{\bar{\alpha}}^2 (N-Z)^{-1} \approx 2\hbar\omega \sum_{\bar{\alpha}} c_{\bar{\alpha}}^2 (N-Z)^{-1}$ from the RPA value of $\Omega_{\mathrm{IAS}} = \epsilon_0^c$. Taking the average value for $2\hbar\omega$ as 20 MeV, an upper limit for $\sum_{\bar{\alpha}} c_{\bar{\alpha}}^2$ as 0.01, and a lower limit for $N-Z$ as 2, for even-even $N \neq Z$ systems, we see that $\delta\Omega$ is not expected to be more than 100 keV. In the truncated TDA, where only the poles at $\Omega - \epsilon_0^c = \epsilon_{\alpha_0}$ are retained, the shift will be twice as large. Kuo[5] has calculated the proton-particle, neutron-hole excitation for ^{208}Pb in the truncated TDA. The result is that in ^{208}Bi the calculated energy of the IAS of the ground state of ^{208}Pb is too *low* by as much as 4 MeV when compared with experimental data, contrary to the positive small shift expected from the discussion given above. This seems to be one example of a good shell-model calculation suffering from a *lack of self-consistency* between the residual interaction and the average sp field.

In reality the ϵ^c are not quite degenerate. This will result in a shift in Ω as well as introduce some isospin impurities into $|\tilde{0}\rangle$ and $B_{\mathrm{IAS}}^\dagger|\tilde{0}\rangle$. This we shall now discuss. We first consider the general nondegenerate case where

$$\epsilon_\sigma^c = \epsilon_0^c + \Delta_\sigma, \quad \sigma = a, \bar{a}. \tag{6}$$

Expanding $g(\Omega)$ in a Taylor series to first order in Δ at $\Omega = \epsilon_0^c$ and $\epsilon_\sigma^c = \epsilon_0^c$ gives

$$\delta\Omega = \left(\sum_\alpha \Delta_\alpha c_\alpha^2 - \sum_{\bar{\alpha}} \Delta_{\bar{\alpha}} c_{\bar{\alpha}}^2\right)(N-Z)^{-1}. \tag{7}$$

For the trivial case when $\Delta_\sigma = \Delta$ we have $\delta\Omega = \Delta$, as expected, since $\sum_\alpha c_\alpha^2 - \sum_{\bar{\alpha}} c_{\bar{\alpha}}^2 \equiv N-Z$. A simple and physical but nontrivial, nondegenerate case occurs when

$$\Delta_{a_0} = 0, \quad -\Delta_{a'} = \Delta_{\bar{a}} = \Delta > 0. \tag{8}$$

The dispersion, $\Delta \sim 1$ MeV, of ϵ^c is small because of the long range nature of the Coulomb interaction. In this case we have

$$\begin{aligned}\delta\Omega &= \Omega_{\mathrm{IAS}} - \epsilon_0^c \\ &= -\Delta\left(\sum_{\alpha'} c_{\alpha'}^2 + \sum_{\bar{\alpha}} c_{\bar{\alpha}}^2\right)(N-Z)^{-1}.\end{aligned} \tag{9}$$

This represents a very small shift because $\sum_{\bar{\alpha}} c_{\bar{\alpha}}^2 \approx \sum_{\alpha'} c_{\alpha'}^2$ is just the HF isospin impurity which is less than 1%.[6]

We now calculate the isospin impurity in $|\tilde{0}\rangle$, which is proportional to the difference $\langle\tilde{0}|T^2|\tilde{0}\rangle - T_0(T_0+1) = \langle\tilde{0}|T_-T_+|\tilde{0}\rangle$, where $T_0 = \frac{1}{2}(N-Z)$. We first note that $|\tilde{0}\rangle$ is obtained from $|0\rangle$ through the canonical transformation[7]

$$|\tilde{0}\rangle = N_0 e^s |0\rangle, \tag{10}$$

where $N_0^2\langle 0|e^{s^\dagger} e^s|0\rangle = 1$, $s = \sum_{\alpha\bar{\alpha}} C_{\alpha\bar{\alpha}} A_\alpha^\dagger A_{\bar{\alpha}}^\dagger$, and the correlation coefficients $C_{\alpha\bar{\alpha}}$ must satisfy $\sum_\alpha x_\alpha^\lambda C_{\alpha\bar{\alpha}} = y_{\bar{\alpha}}^\lambda$, for all $\bar{\alpha}$ and *physical* λ.[7] It then follows that

$$\langle\tilde{0}|T_-T_+|\tilde{0}\rangle = c_\alpha d_{\alpha\bar{\beta}} C_{\beta\bar{\beta}} c_\beta + 2c_\alpha d_{\alpha\bar{\alpha}} c_{\bar{\alpha}} + c_{\bar{\alpha}} c_\alpha + c_{\bar{\alpha}} C_{\alpha\bar{\alpha}} d_{\alpha\bar{\beta}} c_{\bar{\beta}}. \tag{11}$$

where all repeated subscripts are summed over and d is a Hermitian matrix with

$$d_{\alpha\bar{\alpha}} \equiv N_0^2\langle\tilde{0}|e^{s^\dagger} A_\alpha^\dagger A_{\bar{\alpha}}^\dagger e^s|\tilde{0}\rangle = (1-u)_{\alpha\beta}^{-1} C_{\beta\bar{\alpha}}, \tag{12}$$

where $u_{\alpha\beta} \equiv \sum_{\bar{\alpha}} C_{\alpha\bar{\alpha}} C_{\beta\bar{\alpha}}$. It is easy to satisfy oneself that for the case of degenerate ϵ^c, (11) is identically zero, as expected. For the nondegenerate case specified by (8), expanding the x and y amplitudes of the IAS to first order in $\Delta/2\hbar\omega$, we get the important result

$$\langle\tilde{0}|T_-T_+|\tilde{0}\rangle \approx (\Delta/2\hbar\omega)^2 \sum_{\bar{\alpha}} c_{\bar{\alpha}}^2, \tag{13}$$

where we have used the normalization condition $\sum_\alpha x_\alpha^2 - \sum_{\bar{\alpha}} y_{\bar{\alpha}}^2 = 1$ and approximated $u_{\alpha\beta}$ by $\delta_{\alpha\beta} u$.

The result in (14) is to be compared with $\langle 0|T_-T_+|0\rangle = \sum_{\bar{\alpha}} c_{\bar{\alpha}}^2$, in the HF approximation. In other words, the proton-neutron correlation reduces the isospin impurity in the nuclear ground state by a factor of $(\Delta/2\hbar\omega)^2$. This conclusion may depend on the assumption of the factorizability of the isovector residual N-N interaction. However it must be emphasized that neither (11) nor the vanishing of its right-hand side when ϵ^c is degenerate depends on this assumption. Since it is established that the impurity in $|\tilde{0}\rangle$ arises from a small but finite dispersion of ϵ^c, it therefore must at least be of order $\Delta/2\hbar\omega$. The factorizability assumption is perhaps responsible for the right-hand side of (14) being quadratic instead of linear in $\Delta/2\hbar\omega$. At any rate, a significant reduction in the impurity is certain. Previous estimations of the nuclear isospin impurity, without considering the proton-neutron correlation, have already shown that it is quite small, of the order of a fraction of a percent.[6,8]

Finally it can be shown that, for the special nondegenerate case (8), the superallowed Fermi-β^+-decay matrix element does not deviate from the expected value between states of good isospin, i.e.,

$$\langle\tilde{0}|T_+B_{\mathrm{IAS}}^\dagger|\tilde{0}\rangle = c_\alpha x_\alpha + c_\alpha C_{\alpha\bar{\alpha}} d_{\beta\bar{\alpha}} x_\beta - c_\alpha d_{\alpha\bar{\alpha}} y_{\bar{\alpha}} + x_\alpha d_{\alpha\bar{\alpha}} c_{\bar{\alpha}} - c_{\bar{\alpha}} d_{\alpha\bar{\alpha}} C_{\alpha\bar{\beta}} y_{\bar{\beta}}$$
$$\simeq (2T_0)^{1/2} = \langle T_0, T_0|T_+|T_0, T_0-1\rangle, \tag{14}$$

to second order in Δ, in the approximation discussed here.

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Excitation of Abnormal Parity States by α Particles Acting with Velocity-Dependent Central Forces*

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The violation of the normal parity rule in (α,α') scattering is related to a velocity dependence in a purely central, spin-independent α-N force. The distorted-wave Born-approximation formalism of the process is presented, and the specific case of the 3^+ state in Ni^{58} is calculated in the plane-wave Born approximation. The role of the velocity dependence is discussed in physical terms, and a formal similarity with Weber's old theory of electromagnetism is found.

It is well known that in single-stage inelastic scattering the transfer of orbital angular momentum l from the projectile to the target and the parity change $\Delta\pi$ of the target are related by the so-called "normal" parity rule $(-)^l = \Delta\pi$, if the force between the projectile and the target nucleons is static and local, and exchange is ignored. Essentially, this is because the only operator in the multipole expansion of such a force that has the required multipolarity l is Y_l. One particularly striking consequence of this rule is the absolute prohibition of the direct, single-stage excitation of states of abnormal parity from a 0^+ ground state by α particles since in this case there cannot, of course, be any question of spin flip.

Nevertheless, such processes do occur, and as one of several possible explanations a spin-orbit α-N force has been proposed by Eidson and Cramer.[1] Although no calculation has been published to our knowledge, a more complete discussion was given by Satchler[2] who showed that the essential role of the spin-orbit force was to permit the formation of composite tensors through the coupling of the $\vec{L}$ operator with spherical harmonics. Without at all developing the idea he then indicated that a $\vec{p}$ operator (momentum) would do just as well as the $\vec{L}$ operator coming from the spin-orbit force.

In the light of this suggestion, we wish to point out that excitation of abnormal parity states can take place with a central, spin-independent α-N force, provided it is allowed to be velocity dependent. We have, in fact, found that we can get