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Quantum holonomy and link invariants

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It is shown that, in a non-Abelian quantum field theory without an anomaly and broken symmetry, the set of all matrix-valued quantum holonomies $\Psi[\gamma]\equiv\langle P\exp(i\oint_\gamma A\,dx)\rangle$ for closed contours γ form a commutative semigroup, whereas $\langle P\exp(i\int_\alpha A\,dx)\rangle=0$ for every open path α. The eigenvalues $\Phi[\gamma]$ of $\Psi[\gamma]$ are classified according to the irreducible representations of the gauge group. In an irreducible representation ρ, $\mathrm{Tr}(\Psi[\gamma])=\Phi[\gamma]\mathrm{Tr}(1_\rho)$ is a Wilson loop. This equation solves a puzzle in the relation between link invariants and Wilson loops in the Chern-Simons theory in three dimensions when the gauge group is $\mathrm{SU}(N|N)$, and provides useful insight in understanding nonperturbative quantum chromodynamics as a string theory.

In gauge theories [1], probably the most important quantity is the matrix-valued holonomy $P\exp(i\oint_\gamma dx)$, where A is the connection (or vector potential), γ a closed contour, and P means path ordering. In the case of classical electromagnetism, the contour integral gives the magnetic flux, and the holonomy is the celebrated Aharonov-Bohm phase [2], which is itself a special case of Berry's phase [3]. The analogous quantity in quantum field theory is conventionally taken to be the Wilson loop [4] $W[\gamma]=\mathrm{Tr}\langle P\exp(i\oint_\gamma A\,dx)\rangle$, namely, the *trace* of the expectation value of the holonomy. Wilson loops are the keys to a geometric approach to quantum field theories [5] and are essentially the Boltzmann weights in lattice gauge theories [4,6]. Recently, Witten's observation [7] that Wilson loops in the three-dimensional Chern-Simons theory are topological invariants of links has revealed a glimmer of the deep connections among low-dimensional quantum field theory, statistical models, knot theory, and quantum groups that are just beginning to be understood [8].

The Wilson loop rather than the quantum holonomy $\langle P\exp(i\oint_\gamma A\,dx)\rangle$ has been the focus of attention because the former is manifestly gauge invariant while the latter is not. However, the act of taking the trace of a matrix may have masked some nontrivial algebraic property the matrix possesses. Indeed, we shall show in this paper that when gauge anomalies and spontaneous symmetry breaking are absent in a quantum field theory over a compact manifold $\mathcal{M}$, the quantum holonomies form a commutative semigroup valued in the matrix representation of the universal enveloping algebra of the Lie algebra of the gauge group G. We shall give an example that strikingly illustrates the difference between the Wilson loop and the quantum holonomy: information containing the properties of the $\mathrm{SU}(N|N)$ Chern-Simon theory lies in its quantum holonomies, but not in its Wilson loops, which vanishes identically for every γ.

We consider the expectation value of a functional $f(A)$:

$$Z\equiv V^{-1}\int \mathcal{D}A e^{iI[A]}f(A)\,, \tag{1}$$

where $A=A(x)=A_\mu^a(x)t_a$ is the matrix-valued connection (t_a are representations of the generators of G), $I[A]$ a gauge-invariant action, $V=\int\mathcal{D}g$, $g\in G$, and $\mathcal{D}A$ and $\mathcal{D}g\equiv\prod_x d\mu(g(x))$ are invariant measures [9]. Under a local gauge transformation,

$$A(x)\to A(x)^{g(x)}=\Omega_g^{-1}(x)A(x)\Omega_g(x) + \Omega_g^{-1}(x)\partial\Omega_g(x)\,, \tag{2}$$

and under a global gauge transformation,

$$A(x)\to A(x)^g=\Omega_g^{-1}A(x)\Omega_g\,. \tag{3}$$

The set of all global transformations is a subset of the set of all local transformations.

The integral $\int\mathcal{D}A$ in (1) includes integrating over the entire orbit $\{A(x)\}=\{A(x)^{g(x)}|g(x)\in G\}$ for every $A(x)$, while one actually wants an integration including only one point in each orbit. That is one wants to integrate over the equivalence classes $[A(x)]=\{A(x)\}/G$. This is the reason why the right-hand side of (1) is divided by V. This program can be carried out using the method of Faddeev and Popov [10]. Let $F(A^g)$ be a (gauge fixing) function of A such that $F(A^g)=0$ has *exactly one* solution g_0 for every orbit. Define the function $\Delta_F[A]$ by

$$1=\Delta_F[A]\int\mathcal{D}g\,\delta(F(A^g))\,. \tag{4}$$

It follows that $\Delta_F^{-1}[A]$ is also (locally) gauge invariant, which justifies it being written as a function of the equivalence class $[A]$. If the function $f(A)$ in (1) is gauge invariant, then, upon substituting the right-hand side of (4) into (1), it is readily verified that the integrand of $\int \mathcal{D}g$ is independent of g, so that $\int \mathcal{D}g$ cancels the factor V^{-1}, and one obtains the well-known result [10] that

$$Z=\int \mathcal{D}A\,\Delta_F[A]e^{iI[A]}f(A)\delta(F(A^g))$$
$$=\int \mathcal{D}A\,\Delta_F[A]e^{iI[A]}f(A)\delta(F(A)) \tag{5}$$

is precisely the sought for g-independent path integral over $[A]$.

Consider now the case when $f(A)$ is *not* gauge invariant. Specifically consider the matrix-valued exponentiated path integral

$$f(A,\alpha)=P\exp\left[i\int_\alpha A\,dx\right], \tag{6}$$

where P mean path ordering and α is a path from x_0 to x_0'. Under a local gauge transformation [5],

$$f(A,\alpha)\to f(A^g,\alpha)=\Omega_g^{-1}(x_0')f(A,\alpha)\Omega_g(x_0). \tag{7}$$

The fact that the two matrices $f(A,\alpha)$ and Ω_g do not commute appears to prevent one from deriving (5). The standard way to circumvent this apparent difficulty is to replace f by its matrix trace $\mathrm{Tr}(f)$ which, by virtue of the cyclic symmetry of the trace, is locally gauge invariant provided $x_0=x_0'$, i.e., provided α is a closed contour. The corresponding Z is what is known as a Wilson loop.

Let us nevertheless examine the *global* property of the path integral

$$Z(\alpha)_{rs}=V^{-1}\int \mathcal{D}g\int \mathcal{D}A\,\Delta_F[A]$$
$$\times e^{iI[A]}(f(A,\alpha))_{rs}\delta(F(A^g)). \tag{8}$$

Henceforth rs, which are indices of an irreducible matrix representation of G, will be suppressed. The transformation property of $Z(\alpha)$ is revealed by the following maneuver. Let g' be a *global* gauge transformation. Write A in (8) as $A^{g'^{-1}g'}$ and rename $A^{g'^{-1}}$ as A. Use the fact that $\mathcal{D}A$, $\Delta_F[A]$, and the action I are gauge invariant, $\int\mathcal{D}g=\int\mathcal{D}g'g$, and the x-independent version of (7) to reexpress the right-hand side of (8) as

$$V^{-1}\int \mathcal{D}g'g\int \mathcal{D}A\,\Delta_F[A]e^{iI[A]}\Omega_{g'}^{-1}f(A,\alpha)\Omega_{g'}\delta(F(A^{g'g}))$$
$$=\Omega_{g'}^{-1}Z(\alpha)\Omega_{g'}. \tag{9}$$

Since this is true for every global transformation, one has $[Z(\alpha),\Omega_{g'}]=0$, $\forall g'\in G$, and, from Schur's Lemma,

$$Z(\alpha)=C(\alpha)1_\rho, \tag{10}$$

where the scalar function $C(\alpha)$ is any one of the diagonal matrix elements of $Z(\alpha)$ and 1_ρ is the matrix representation of the identity element in G in the irreducible representation ρ.

Equation (10) suggests that $Z(\alpha)$ belongs to some algebraic structure. Let $X_1,\ldots,X_n$ be a basis of the Lie algebra $\mathcal{G}$ of Lie group G in the representation ρ. Then $f(A,\alpha)$ is a linear combination of the set of monomials $\{X_1^{v_1}X_2^{v_2}\cdots X_n^{v_n}|v_1\geq 0,\ldots,v_n\geq 0\}$, that is, it is valued in the ρ representation of the universal enveloping algebra $\mathcal{U}(\mathcal{G})$ [the representation of $\mathcal{G}$ extends to $\mathcal{U}(\mathcal{G})$ by the Poincaré-Birkhoff-Witt theorem [11]]. The path integration in (8) affects only the coefficients of the $f(A,\alpha)$ as a linear combination of the monomials. Therefore $Z(\alpha)$ is also a linear combination of the monomials. Therefore $Z(\alpha)\in\rho(\mathcal{U}(\mathcal{G}))$. An infinitesimal global tansformation has the form $\Omega_\epsilon=1+\epsilon_iX_i$. $Z(\alpha)$ commuting with all such Ω_ϵ's implies that it commutes with all the X_i's, and therefore with all matrices in $\rho(\mathcal{U}(\mathcal{G}))$. We conclude that $Z(\alpha)$ is valued in the ρ representation of the center of $\mathcal{U}(\mathcal{G})$. This also gives another proof of (10).

We now analyze the consequence of changing the integration variable A in (8) to $A^{g'}$ by a *local* gauge transformation. From (7) and (10),

$$Z(\alpha)[\Omega_{g'}(x_0')-\Omega_{g'}(x_0)]=0. \tag{11}$$

This being true for every local gauge transformation g', and $\Omega_{g'}(x_0')$ being generally not equal to $\Omega_{g'}(x_0)$ when $x_0'\neq x_0$, one concludes that

$$C(\alpha)\begin{cases}=0, & \alpha \text{ is an open path},\\ \neq 0, & \alpha \text{ is a closed contour}.\end{cases} \tag{12}$$

Equations (10) and (12) form the main result of this paper. In what follows we reserve α to represent an open path and use γ to denote a *closed* contour. The result $C(\alpha)=0$ may be understood as follows. Because α is an open path, $f(A,\alpha)$ has a strong dependence on the geometry of α. Changing A in $f(A,\alpha)$ is like changing the geometry of α with only its end points fixed. Therefore the path integral in (8) is like a sum of $f(A,\alpha)$ over paths of random geometries but fixed end points, and $Z(\alpha)$ is like a sum of *random* matrices.

To see that $C(\gamma)$ does not depend on the initial point x_0 (which is also the final point) of γ, note that the gauge dependence of A^g in the factor $\delta(F(A^g))$ in (8) can be absorbed by $\mathcal{D}A$ so that, for a closed contour, (8) may be rewritten in the form of (5), with $f(A)$ there replaced by the *gauge-invariant* quantity

$$\tilde{f}(A,\gamma)\equiv V^{-1}\int\mathcal{D}g\,\Omega_g(x_0)f(A,\gamma)\Omega_g^{-1}(x_0)$$
$$=\int d\mu(g)f(A^{g^{-1}},\gamma)\Big/\int d\mu(g)$$
$$=\int d\mu(g)f(A^{g},\gamma)\Big/\int d\mu(g).$$

The second equality being a consequence that only the gauge transformation at x_0 is involved. A proof of (10) again follows. Moreover, this relation defines $C(\gamma)$ as a conjugacy class, that is, $aC(\gamma)a^{-1}=C(\gamma)$, $\forall a\in\rho(G)$. If γ' and γ differ only by their initial points, then $\tilde{f}(A,\gamma')=a\tilde{f}(A,\gamma)a^{-1}$ for some $a\in\rho(G)$. It follows that $C(\gamma')=aC(\gamma)a^{-1}=C(\gamma)$. Therefore $C(\gamma)$ is initial-point independent.

Henceforth, we denote the path integration on the right-hand side of (8) by $\langle f(A,\alpha)\rangle$. For a closed contour, we reexpress the result as

$$\Psi[\tau(\gamma)]\equiv\left\langle P\exp\left[i\oint_\gamma A\,dx\right]\right\rangle=\Phi[\gamma]1_\rho, \tag{13}$$

which also defines the matrix function Ψ and the scalar function Φ; $\tau(\gamma)$ will be explained later. It is sufficient for the formal relation (13) to hold when the following two conditions are satisfied. (a) The theory does not have any gauge anomaly. Implicit in this condition is that, if one were to study perturbation theory, a regularization that preserves gauge invariance would exist. (b) None of the symmetries of the theory is spontaneously broken. In this case a heuristic argument for (13) is as follows. With no symmetry spontaneously broken, the expectation value in (13) may be viewed as that of a vacuum state that has zero quantum number for every symmetry of the theory. Then the only operators that can have nonvanishing vacuum expectation values are those that transform as scalars with respect to the symmetries of the theory. The result $\langle P\exp(i\int_\alpha A\,dx)\rangle=0$ may be interpreted the same way: because of (7), no nonvanishing linear combination of $P\exp(i\int_\alpha A\,dx)$ can ever transform as a scalar. For closed contours, we believe the quantum holonomy to be as well defined as the Wilson loop.

We now explain the meaning of the notation $\tau(\gamma)$. The quantity $\langle P\exp(i\oint_\gamma A\,dx)\rangle$ is valued on the fiber bundle whose base is $\mathcal{M}$ and whose fiber is G. Whereas γ is a closed loop in $\mathcal{M}$, $\tau(\gamma)$ is the lift of γ into the fiber bundle; it has two open ends that carry the "colors" [the indices r, s in (8)] of G. Equation (13) states that [for gauge theories that meet the conditions (a) and (b)] colors are conserved on $\tau(\gamma)$ and the Berry's phase is classified according to the irreducible representation of G so that all color states in a given irreducible representation have the same Berry's phase. Thus the quantum holonomies $\Psi[\tau(\gamma)]$ of a gauge theory form a *commutative semigroup*. It is a semigroup because every $\Psi[\tau(\gamma)]$ does not have an inverse. The composition law of $\Psi[\tau(\gamma)]$ is just the commutative composition law of open strings, or that of a commutative groupoid [12] (roughly speaking, a groupoid is a set of oriented open strings with an associative composition that multiplies two strings by connecting them tip to end), which, in $\mathcal{M}$, becomes the composition of contours:

$$\Psi[\tau(\gamma_2)]\Psi[\tau(\gamma_1)]=\Psi[\tau(\gamma_1)]\Psi[\tau(\gamma_2)]=\Psi[\tau(\gamma_2)\circ\tau(\gamma_1)]=\Psi[\tau(\gamma_1)\circ\tau(\gamma_2)]=\Psi[\tau(\gamma_1\circ\gamma_2)]=\Psi[\tau(\gamma_2\circ\gamma_1)]\,.$$

When $\mathrm{Tr}(1_\rho)\neq0$, the value of the Wilson loop $W[\gamma]=\mathrm{Tr}\{\Psi[\tau(\gamma)]\}=\Phi(\gamma)\mathrm{Tr}(1_\rho)$ differs from $\Phi(\gamma)$ by a nonessential proportionality constant. This is no longer true when $\mathrm{Tr}(1_\rho)=0$. Then the Wilson loop vanishes identically and the information of the quantum theory is contained only in $\Phi[\gamma]$. One such example will be discussed below. In any case, it is readily seen from (13) that $\Psi[\tau]$ retains all the useful properties of the Wilson loop. For instance, the quantum theory of loops [5] can just as well be applied to $\Psi[\tau]$.

There is at least one theory that satisfies criteria (a) and (b) given above, namely, the topological Chern-Simons theory (CST) in three dimensions. In this case, the integration contours may be knots or links, which we shall denote collectively by K. Witten [7] has shown that the Wilson loop $W[K]$ in three-dimensional (3D) CST (for certain quantized values of the coupling constant) must be a link invariant [13], and that the partition function of the theory is a topological invariant of three-manifolds. Both of these assertions have been verified by direct computations [14,15]. This implies that for 3D CST the path integral is well defined. Since $\Psi[\tau(K)]$ must satisfy the same skein relation as its trace, it follows from (13) that $\Phi[K]$ is a link invariant. One may visualize $\tau(K)$ as a one-tangle equivalence class, namely, the class of topological objects with two fixed open ends whose closure is the link K. The set of all $\Psi[\tau(K)]$'s forms a semigroup instead of a group because generally a one-tangle does not have an inverse under the groupoid composition. This is reflected in the fact that a link invariant, which is a rational function, generally does not have an inverse that is also a rational function.

A theorem on functors of one-tangles and links on the quantum group, or *quantized* universal enveloping algebra [16,17], having the form

$$\mathcal{V}[\tau(K)]=Q[K]1_\pi\,, \tag{14}$$

has recently been proven [18,19], where $\mathcal{V}[\tau(K)]$ is a one-tangle invariant, $Q[K]$ a link invariant, and 1_π is the identity element of the quantum group in the irreducible representation π. Given Witten's theory on the relation between Wilson loops and link invariants, it would be surprising if only one of (13) and (14) existed. However, we have nothing to say about how the universal enveloping algebra discussed in conjunction with (13) became quantized in the case of 3D CST. Denote isotopy by $\sim$. The implication from (14) that $\mathcal{V}[\tau(K)]=\mathcal{V}[\tau'(K)]$ even when $\tau(K)\not\sim\tau'(K)$ is the counterpart to $\Psi[\tau(K)]$ being independent of the initial-point on K. The striking similarity between (13) and (14) has motivated the conjecture that topological invariants in CST and in quantum groups belong to the same category of functors [19]. The functors $\mathcal{V}$ and Ψ give a stronger relation among the tangle and link invariants than there exists between tangles and links. Denote the closure of τ by $\hat\tau$; $\hat\tau$ is a link. Let $\hat\tau_1\sim\hat\tau_2$, $\tau_1\not\sim\tau_2$, τ_3 be any other one-tangle and $\tau'=\tau_1\circ\tau_3$, $\tau''=\tau_2\circ\tau_3$. Then $\mathcal{V}[\tau']=\mathcal{V}[\tau'']=Q[\hat\tau']1_\pi=Q[\hat\tau'']1_\pi$, although $\hat\tau'\not\sim\hat\tau''$. The same is true when $\mathcal{V}$ is replaced by Ψ and Q by Φ.

Skein relations for Wilson-loop link invariants of CST with other gauge groups have also been derived [20]. As far as (13) and (14) are concerned, the most interesting of these results is that, in the fundamental representation of $G=\mathrm{SU}(N|M)$, $W[K]$ is the HOMFLY polynomial [21] when $N\neq M$, but vanishes identically when $N=M$. On the other hand, the HOMFLY polynomial itself becomes the Alexander-Conway polynomial [22] when $N=M$. This apparent paradox is resolved by (13). Recall that for $\mathrm{SU}(N|M)$, Tr is a *supertrace* [23], so $\mathrm{Tr}(1_\rho)=N-M$ and $W[K]=\Phi[K](N-M)$. Thus, although the bosonic fields and the fermionic fields have the same quantum holonomies, they contribute to the Wilson loop with opposite signs. Consequently in the balanced case with $N=M$, the information of the theory resides not in the Wilson loop which vanishes exactly for every γ, but in the quantum holonomy $\Phi[\gamma]$ which is equal to the Alexander-

Conway polynomial. This structure has been shown to duplicate via (14) on the quantum groups $gl(N+M;N)_{q,s}$ at $q^2=-1$ [19] and $sl(N|M)_s$ [24], which are isomorphic.

Our result seems to suggest that the quantum holonomy defined by a quantum field theory may exist as an abstract semigroup living in the center of $\mathcal{U}(\mathcal{G})$. Whether this is true remains to be seen. In the case of 3D CST, whose apparent relations to quantum groups have already been noted [25], relations (13) and (14) supply another explanation for the integrability of the theory: each quantum holonomy may be interpreted as a conserved *topological charge* of the theory, and there are an infinite number of such conserved charges associated with the infinite number of links. It is worth pointing out that since the condition (a) mentioned above is only a sufficient one, it is not impossible for (13) to hold even in the presence of anomalies. An intriguing possibility is quantum electrodynamics (QED) [26], which has a gauge anomaly [27], but for which the normal Wilson loop (with trace) is a well-defined object. We therefore expect (13) to hold for QED. A more interesting case is quantum chromodynamics (QCD), whose potential gauge anomaly is cancelled owing to flavor symmetry. We therefore expect (13) to hold for QCD as well. Assuming that to be the case, then the set $\{\Psi[\gamma]|$ all classes of $\gamma's\}$ completely defines the theory nonperturbatively as a theory of strings (or of loops in the base manifold $\mathcal{M}$) with Abelian multiplication. In view of the recent interest in studying nonperturbative QCD as a string theory [28], it would be extremely useful to know whatever additional algebraic structure the set may possess.

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