PHYSICAL REVIEW D VOLUME 24, NUMBER 11 1 DECEMBER 1981

Calculation of electroproduction of W bosons in electron-proton collisions in the Weizsäcker-Williams approximation

A. N. Kamal
Theoretical Physics Institute and Department of Physics, University of Alberta, Edmonton, Alberta, Canada

J. N. Ng
TRIUMF, University of British Columbia, Vancouver, British Columbia, Canada

H. C. Lee
Atomic Energy of Canada Ltd., Chalk River, Ontario, Canada
(Received 2 June 1981)

Using the Weizsäcker-Williams equivalent-photon spectrum we calculate the electroproduction cross section for $W^{\pm}$ bosons in electron-proton collisions.

I. INTRODUCTION

Several proposals[1] have been made for electron-proton colliding-beam facilities around the world. Among the interesting physics that will be possible at these facilities of the future is the prospect of electroproduction of $W^{\pm}$ and Z^0. Rough estimates for the electroproduction of these gauge bosons have been made and appear in the proposals[1] for these machines. It is desirable to have a more precise calculation of the production cross sections. In the present paper we have calculated the production cross section for $e+p\to e+W^{\pm}+X$ using the Weizsäcker-Williams[2] approximation for the photon spectrum. We expect the Weizsäcker-Williams approximation to be quite reliable for the electroproduction of $W^{\pm}$ since the vector boson cannot be produced at the lepton vertex in $e+p\to e+W^{\pm}+X$. However, since Z^0 can be produced at the lepton vertex, we do not expect the Weizsäcker-Williams approximation to be reliable for $e+p\to e+Z^0+X$. For this reason we confine our calculation to the electroproduction of $W^{\pm}$ only.

The calculation of $e+p\to e+W^{\pm}+X$ proceeds in the following steps. (1) We calculate photoproduction of $W^{\pm}$ on quarks, $\gamma+q\to W^{\pm}+q$ (see Fig. 1). (2) We fold in the quark distribution functions to generate $\gamma+p\to W^{\pm}+X$. We make use of two quark distribution functions: those of Barger and Phillips[3] and Buras and Gaemers.[4] (3) As a final step we fold in the Weizsäcker-Williams photon spectrum to calculate the cross section for $e+p\to e+W^{\pm}+X$.

In the process of evaluating step (1) we reproduce the results of Mikaelian,[5] thus providing a useful check on that calculation. In Sec. II we describe our calculation, provide the essential details, and present the results. A brief discussion follows in Sec. III.

II. CALCULATION

A. Photoproduction on quarks

The process we first calculate is $\gamma(k)+q(p)\to W^{\pm}(k')+q(p')$. The three processes contributing to the photoproduction on quarks are shown in Fig. 1. This part of the calculation essentially reproduces Mikaelian's[5] results. The final result for the differential cross section for $W^{\pm}$ production on quarks is (see Ref. 5 for details)

$$\begin{aligned}\frac{d\sigma^{\pm}}{dt}&=\frac{\alpha g^2}{16s^2}T(\kappa,\pm Q,s,t)\,,\\ s&=(p+k)^2\,,\ t=(k-k')^2\,,\ \text{and}\ u=M_W{}^2-s-t\,,\\ g^2&=\frac{M_W{}^2G_F}{\sqrt{2}}\cos\theta_C\,,\quad \text{for}\ \Delta S=0\\ &=\frac{M_W{}^2G_F}{\sqrt{2}}\sin\theta_C\,,\quad \text{for}\ \Delta S=1\,.\end{aligned}\tag{2.1}$$

κ is the anomalous magnetic moment of the W which in the standard Weinberg-Salam-Glashow theory is 1.

The form of $T(\kappa,Q,s,t)$ is[5]

$$\begin{aligned}T(\kappa,Q,s,t)=&-16(Q-1)^2\frac{s}{u}-16Q^2\frac{u}{s}-32Q(Q-1)t\frac{M_W{}^2}{su}+16\left[\frac{(Q-1)}{u}-\frac{Q}{s}\right]\left[\frac{2tM_W{}^2-(1+\kappa)su}{M_W{}^2-t}\right]\\ &-\frac{8t}{M_W{}^2}+8\left[\frac{2s}{M_W{}^2}(s+t)+(1+\kappa)\left(t-\frac{(s+t)^2}{M_W{}^2}\right)\right]\Big/(M_W{}^2-t)\\ &-2\left[8s^2-16tM_W{}^2-4(1+\kappa)s^2\left(1+\frac{t}{M_W{}^2}\right)+(1+\kappa)^2\left(4su+\frac{(s^2+u^2)t}{M_W{}^2}\right)\right]\Big/(M_W{}^2-t)^2\,.\end{aligned}\tag{2.2}$$

B. Photoproduction on proton

The process we calculate next is $\gamma+p\to W^{\pm}+X$. If we assume that the struck quark has a momentum fraction x and the final hadron invariant mass is m_f^2, then the quark-parton model leads to a double distribution (note that $s\to xs$, $t\to t$, and $u=M_W^2-t-xs$)

$$\frac{d\sigma^{\pm}}{dm_f^2dt}=\frac{\alpha}{16s^2t}\sum_i g_i^2 f_i(x)T(\kappa,\pm Q_i,xs,t)\,, \tag{2.3}$$

where $f_i(x)$ is the ith quark distribution density and $x=t/(t+m^2-m_f^2)$ with $m=$ proton mass. Written out explicitly in terms of the u, d, and s quarks, one gets

$$\frac{d\sigma^{+}}{dm_f^2dt}=\frac{\alpha G_F M_W^2}{16\sqrt{2}s^2t}\{u(x)T(\kappa,\tfrac{2}{3},xs,t)+[\bar{d}(x)\cos^2\theta_C+\bar{s}(x)\sin^2\theta_C]T(\kappa,\tfrac{1}{3},xs,t)\}\,, \tag{2.4}$$

$$\frac{d\sigma^{-}}{dm_f^2dt}=\frac{\alpha G_F M_W^2}{16\sqrt{2}s^2t}\{\bar{u}(x)T(\kappa,\tfrac{2}{3},xs,t)+[d(x)\cos^2\theta_C+s(x)\sin^2\theta_C]T(\kappa,\tfrac{1}{3},xs,t)\}\,. \tag{2.5}$$

The last two equations confirm the correctness of the calculation of Ref. 5.

Next, we generate $d\sigma^{\pm}/dm_f^2$ by integrating over the allowed range of t,

$$M_W^2-2E(E'+|\vec{k}'|)\le t\le M_W^2-2E(E'-|\vec{k}'|)\,, \tag{2.6}$$

where $(E,\vec{k})$ and $(E',\vec{k}')$ are the energy-momentum of the photon and the $W^{\pm}$, respectively. In the γp center-of-mass frame

$$E=\sqrt{s}/2\,,$$
$$E'=(s-\Delta)/2\sqrt{s}\,, \tag{2.7}$$
$$\Delta=m_f^2-M_W^2\,.$$

The resulting $d\sigma^{\pm}/dm_f^2$ for $s=2.5\times10^4$ GeV^2 are shown in Fig. 2. The cross section rises steeply near the end point of the spectrum $m_f^2=6907$ GeV^2 ($M_W=75$ GeV is used in all calculations), and then drops precipitously. This occurs over such a small measure of m_f^2 that it hardly contributes significantly to $\sigma^{\pm}(s)$. The rise of $d\sigma^{\pm}/dm_f^2$ near the threshold in m_f^2 is quite smooth and is not plotted out. The variation of $d\sigma^{\pm}/dm_f^2$ near the end point is exaggerated because of the use of a logarithmic scale for m_f^2.

The final integration over m_f^2 yields $\sigma^{\pm}(s)$. In Fig. 3 we show $\sigma^{\pm}(s)$ vs s for the Barger-Phillips[3] and the Buras-Gaemers[4] quark distribution functions.

We notice that the general trend of $\sigma^{\pm}(s)$ vs s is the same for both parametrizations and the differences are only slight. The general trend is for the Buras-Gaemers[4] distribution functions to generate slightly lower cross sections. The photoproduction cross section for $W^{\pm}$ with $\kappa=+1$ is significantly higher than the cases for $\kappa=0$ and -1. The difference is large enough to prove to be a useful tool in discriminating the standard Weinberg-Salam-Glashow theory ($\kappa=+1$) from other theories. We note that $\sigma^{\pm}(s)$ calculated here are very similar to those obtained in Ref. 5, where a different set of distribution functions were used.

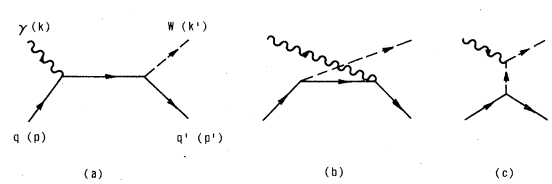

FIG. 1. The three diagrams for $\gamma+q\to W+q$.

C. Electroproduction on proton

As the last step in our calculation of the electroproduction cross section for $e+p\to e+W^{\pm}+X$, we use the Weizsäcker-Williams[2,6] equivalent-photon spectrum and generate the electroproduction cross section. Using M^2 as the ep center-of-mass (energy)^2 one gets

$$\sigma^{\pm}_{ep}(\kappa,M^2)=\frac{\alpha}{\pi}\ln\frac{M^2}{m_e^2}\int_{M_W^2}^{M^2}\frac{ds}{s}\left(1-\frac{s}{M^2}+\frac{s^2}{2M^4}\right)\sigma^{\pm}_{\gamma p}(\kappa,s)\,. \tag{2.8}$$

The result of this integration for the Barger-Phillips[3] and the Buras-Gaemers[4] parametrizations of the quark distribution functions is shown in Fig. 4. As the quantum-chromodynamic corrections soften the quark distribution functions, i.e., deplete large-x quarks, and large-x quarks are more efficient in producing W's, the Buras-Gaemers quark distribution functions lead to smaller production cross sections.

III. DISCUSSION

Estimates for $\sigma_{ep}(M^2)$ have been made in the past (see, in particular, the CHEEP report[1]. Basing their argument on several plausible quark model assumptions they estimate (with $M_W=65$ GeV)

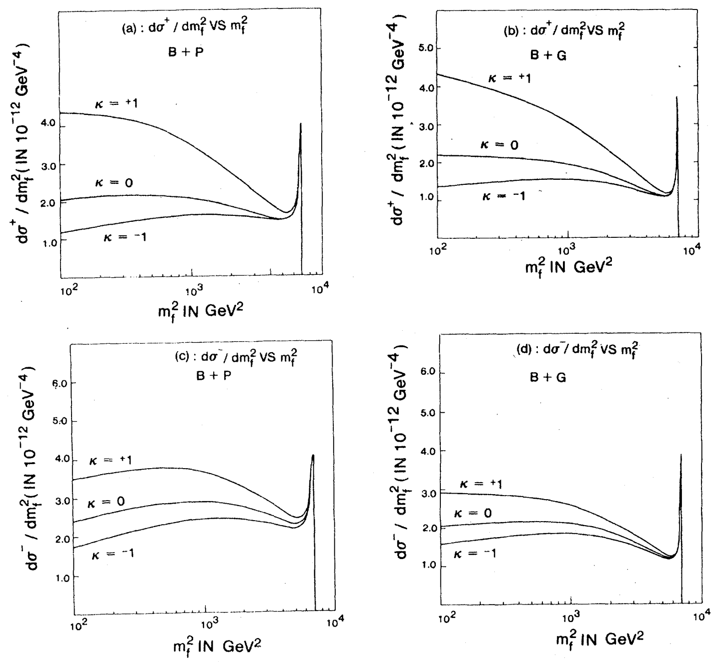

FIG. 2. $d\sigma^{\pm}/dm_f{}^2$ vs $m_f{}^2$ for $\gamma+p\to W^{\pm}+X$ for Barger-Phillips (marked B+P) and Buras-Gaemers (marked B+G) quark distribution functions at s [invariant γp (energy)^2] $=2.5\times10^4$ GeV^2.

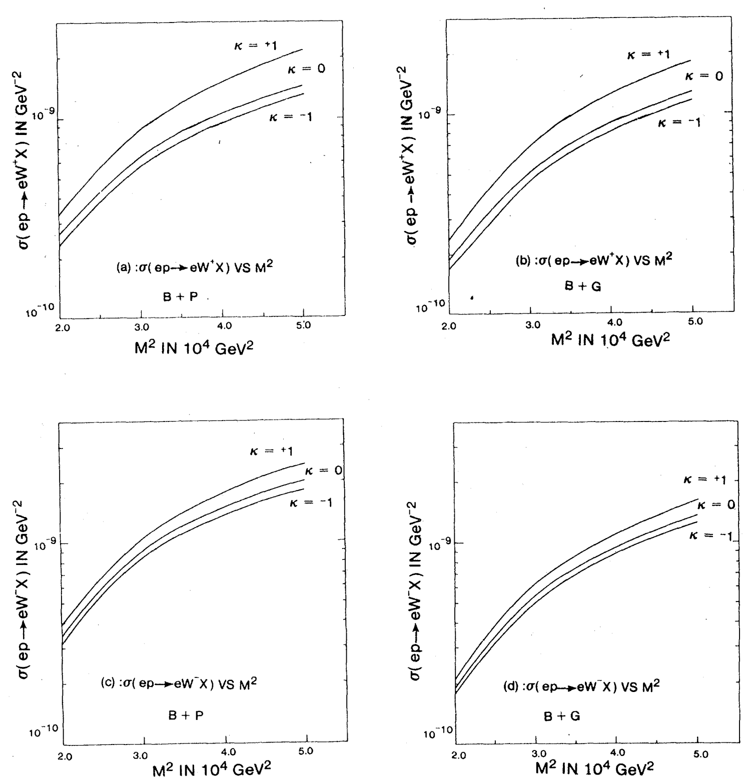

FIG. 3. $\sigma^{\pm}(s)$ vs s for $\gamma+p\to W^{\pm}+X$ using Barger-Phillips (B+P) and Buras-Gaemers (B+G) quark distribution functions. Multiply σ by 4×10^{-28} to convert to cm^2.

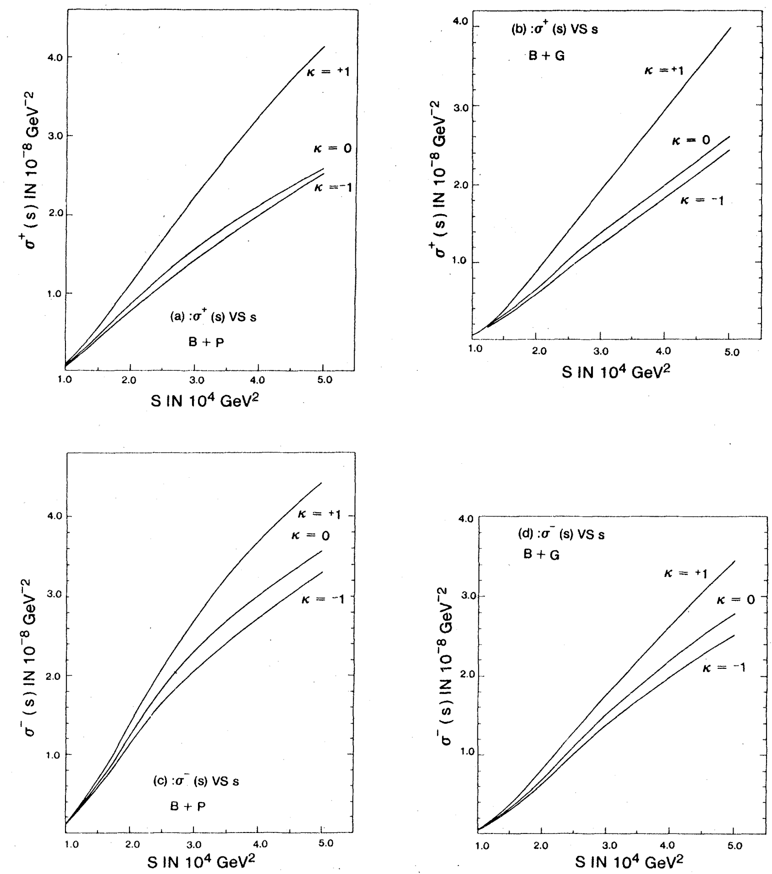

FIG. 4. $\sigma(ep \to eWX)$ vs M^2 [invariant ep (energy)^2] for Barger-Phillips (B+P) and Buras-Gaemers (B+G) quark distribution functions. Multiply σ by 4×10^{-28} to convert to cm^2.

$$\sigma^{+}_{ep} = 2\times 10^{-38}\ \text{cm}^2\,, \qquad \sigma^{-}_{ep} = 5\times 10^{-38}\ \text{cm}^2 \tag{3.1}$$

at $M^2 = 2.7\times 10^4$ GeV^2. Note that for energetic head-on collision of electrons with protons, $M^2 \simeq 4E_e E_p$, so that the head-on collision of a 10-GeV electron with a 500-GeV proton results in $M^2 \simeq 2\times 10^4$ GeV^2. At the same value of M^2 we find that our estimates are higher. We find (with $M_W = 75$ GeV)

$$\sigma^{+}_{ep} = \begin{cases} 2\times 10^{-37}\ \text{cm}^2 & \text{(Buras-Gaemers)} \\ 2.4\times 10^{-37}\ \text{cm}^2 & \text{(Barger-Phillips)}\,, \end{cases} \tag{3.2}$$

$$\sigma^{-}_{ep} = \begin{cases} 2\times 10^{-37}\ \text{cm}^2 & \text{(Buras-Gaemers)} \\ 3.0\times 10^{-37}\ \text{cm}^2 & \text{(Barger-Phillips)}\,. \end{cases} \tag{3.3}$$

These estimates are higher than those in the CHEEP report[1] by an order of magnitude for σ^{+}_{ep} and a factor of 4 for σ^{-}_{ep}. With a machine luminosity of 10^{32} cm^{-2}sec^{-1} one would generate an event rate of 1–2 produced $W^{\pm}$ per day. The event rate goes up by a factor of 2 at $M^2 = 5\times 10^4$ GeV^2. Note that this is only a part of the W-production mechanism. The estimate of the CHEEP report[1] for $e+p \to \nu + W^{\pm} + X$ is $\simeq 10^{-37}$ cm^2 which will essentially double the event rate.

The detection of $W^{\pm}$ will involve wide-angle leptons through $W^{\pm} \to \mu^{\pm} + \nu_\mu$ and $e^{\pm} + \nu_e$. The branching ratio into either of these modes is (see, for example, the CHEEP report[1])

$$\frac{\Gamma(W^{+} \to e^{+} + \nu_e)}{\Gamma(W^{+} \to \text{all})} \simeq \frac{1}{4N_D}\,, \tag{3.4}$$

where N_D = number of lepton generations assumed to equal the number of quark generations. With $N_D = 3$ this branching ratio is 8%. The rate at which $W^{\pm}$ would be observed to decay into one of the leptonic modes will be an order of magnitude lower than the rate at which they will be produced. We acknowledge that the Weizsäcker-Williams approximation overestimates the cross section and also that a complete calculation would require the photoproduction amplitudes for $k^2 \neq 0$ where we also expect the Z^0-exchange processes to be important.

ACKNOWLEDGMENTS

We wish to thank Bob Teshima for the numerical work. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

APPENDIX A: DETAILS OF CALCULATION

Some of the details of the calculation are provided below. The Barger-Phillips[3] parametrization of quark distribution functions is

$$u(x) = 0.594x^{-1/2}(1-x^2)^3 + 0.461x^{-1/2}(1-x^2)^5 + 0.621x^{-1/2}(1-x^2)^7 ,$$

$$d(x) = 0.072x^{-1/2}(1-x^2)^3 + 0.206x^{-1/2}(1-x^2)^5 + 0.621x^{-1/2}(1-x^2)^7 ,$$

$$\bar{u}(x) = \bar{d}(x) = s(x) = \bar{s}(x) = 0.145x^{-1}(1-x)^9 .$$

The Buras-Gaemers[4] parametrization of quark distribution functions is

$$u(x,\bar{s}) = \frac{3}{B[\eta_1(\bar{s}), 1+\eta_2(\bar{s})]} x^{\eta_1(\bar{s})-1}(1-x)^{\eta_2(\bar{s})} - \frac{1}{B[\eta_3(\bar{s}), 1+\eta_4(\bar{s})]} x^{\eta_3(\bar{s})-1}(1-x)^{\eta_4(\bar{s})} ,$$

$$d(x,\bar{s}) = \frac{1}{B[\eta_3(\bar{s}), 1+\eta_4(\bar{s})]} x^{\eta_3(\bar{s})-1}(1-x)^{\eta_4(\bar{s})} ,$$

where

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

is the Euler beta function,

$$\eta_1(\bar{s}) = 0.70 - 1.1G\bar{s} ,$$

$$\eta_2(\bar{s}) = 2.60 + 5.0G\bar{s} ,$$

$$\eta_3(\bar{s}) = 0.85 - 1.5G\bar{s} ,$$

$$\eta_4(\bar{s}) = 3.35 + 5.1G\bar{s} ,$$

$$G = \tfrac{4}{25}, \quad \text{for four flavors}, \quad \bar{s} = \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} ,$$

$$Q_0^2 = 1.8 \text{ GeV}^2 , \quad \Lambda^2 = 0.09 \text{ GeV}^2 ,$$

$$\bar{u}(x,\bar{s}) = \bar{d}(x,\bar{s}) = s(x,\bar{s}) = \bar{s}(x,\bar{s}) = \tfrac{1}{6}A(\bar{s})x^{-1}(1-x)^{\eta_5(\bar{s})} .$$

For computational purposes we fitted a quadratic form to $A(\bar{s})$ and $\eta_5(\bar{s})$:

$$A(\bar{s}) = 1.21 + 1.4\bar{s} + 0.6\bar{s}^2 ,$$

$$\eta_5(\bar{s}) = 10.0 + 6.6\bar{s} + 1.8\bar{s}^2 .$$

APPENDIX B: END-POINT ANALYSIS

The rise in $d\sigma/dm_f^2$ near the end point in m_f^2

$$m_{f,\max}^2 = (\sqrt{s} - M_W)^2 \tag{B1}$$

is analyzed in this appendix. The energy denominators for Figs. 1(a) and 1(c) are of order M_W^2. As $m_f^2 \to m_{f,\max}^2$, the value of t approaches its kinematic limit

$$t \to t_0 = -M_W(\sqrt{s} - M_W) . \tag{B2}$$

In this limit one can show that

$$u = m_W^2 - t = (st)/(t + m^2 - m_f^2) = \frac{M_W}{M_W - \sqrt{s}} m^2 . \tag{B3}$$

Here m is the proton mass. (B3) implies that $u \ll M_W^2$ and, therefore, near the end point Fig. 1(b) dominates the physics. Near the end point let us set

$$m_f^2 = m_{f,\max}^2 - 2\delta^2 . \tag{B4}$$

Then the range of t is

$$t_0 - (2M_W\sqrt{s})^{1/2}\delta \le t \le t_0 + (2M_W\sqrt{s})^{1/2}\delta . \tag{B5}$$

The contribution to the cross section $d\sigma/dm_f^2$ from Fig. 1(b) is of the form

$$-\int \frac{dt}{tu} \approx \frac{t_0 + m^2 - m_f^2}{t_0} \int \frac{dt}{t^2 + bt + c} , \tag{B6}$$

where

$$b = [2M_W(\sqrt{s} - M_W) + m^2 + 2\delta^2] , \tag{B6}$$

$$c = M_W^2[s - 2M_W\sqrt{s} + M_W^2 - m^2 - 2\delta^2] . \tag{B7}$$

The evaluation of (B6), apart from the prefactor $(t_0 + m^2 - m_f^2)/t_0$, yields

$$\begin{aligned} &\frac{1}{(2M_W\sqrt{s})^{1/2}\delta} \ln(m^2/8\delta^2) , \quad M_W\sqrt{s} \gg \delta^2 \gg m^2 , \\ &\frac{1}{(3M_W\sqrt{s})^{1/2}m} \ln\left(\frac{\sqrt{3/2}-1}{\sqrt{3/2}+1}\right) , \quad \delta^2 = m^2 , \\ &\frac{1}{(2m_W\sqrt{s})^{1/2}m}\left(-\frac{4\delta}{m}\right) , \quad \delta^2 \ll m^2 . \end{aligned} \tag{B8}$$

Clearly this integral goes to zero as $\delta \to 0$ but does have a rise as δ decreases through the region $\delta^2 \gg m^2$. The plots of Fig. 2 exaggerate the rise and fall of $d\sigma/dm_f^2$ near the end point in m_f^2 due to the use of a logarithmic scale for m_f^2.

[1]CHEEP proposal, edited by J. Ellis, B. H. Wiik, and K. Hübner, Report No. CERN 78-02, Intersecting Storage Rings Division (unpublished); HERA proposal, European Committee for Future Accelerators (ECFA), Report No. DESY HERA 80/01 (unpublished); CHEER proposal, Institute of Particle Physics (IPP), Canada report, 1980 (unpublished); TRISTAN proposal, edited by T. Kamae, Y. Shimizu, and M. Igarashi, University of Tokyo Report No. UTPN-165, 1980 (unpublished).

[2]K. F. Weizsäcker, Z. Phys. 88, 612 (1934); E. J. Williams, Phys. Rev. 45, 729 (1934); Kgl. Danske Videnskab. Selskab, Mat-fys. Medd. 13, #4 (1935).

[3]V. Barger and R. J. N. Phillips, Nucl. Phys. B73, 269 (1974).

[4]A. J. Buras and K. J. F. Gaemers, Nucl. Phys. B132, 249 (1978).

[5]K. O. Mikaelian, Phys. Rev. D 17, 750 (1978).

[6]R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957); V. N. Gribov, V. A. Kolkunov, L. B. Okun, and V. M. Shekhter, Zh. Eksp. Teor. Fiz. 41, 1839 (1961) [Sov. Phys.—JETP 14, 1308 (1962)]; Min-Shih Chen and P. Zerwas, Phys. Rev. D 12, 187 (1975); H. A. Olsen, *ibid*. 19, 100 (1979).