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PHYSICAL REVIEW C

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Blocking Effect in the Nuclear Pairing Theory: An Application to the Sn Isotopes

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The effect of blocking, for odd systems, is included in the gap equations of the nuclear pairing theory for a general two-body interaction. These equations are applied to calculate the one-quasiparticle low-energy spectra, the neutron separation energies, and the "odd-even mass differences" for the Sn isotopes. The apparent inflation of the effective pairing strengths in calculations in which the blocking effect is ignored is pointed out.

The Bardeen-Cooper-Schrieffer¹ (BCS) theory of superconductivity has been widely applied to treat the pairing correlation in deformed² as well as spherical³ nuclear systems. As was pointed out by Soloviev,² the BCS trial wave function is suitable only for even systems. For an odd system, the "blocking effect" induced by the unpairing particle should be taken into account. The resulting gap equations, when the pairing interaction is assumed to be the constant pairing force, have previously been derived.^{2,3} In this paper we give the gap equations for a general two-body interaction. Using a realistic nucleon-nucleon residual interaction we apply these equations to the Sn isotopes. The low-energy spectra of the odd-mass isotopes are reasonably well reproduced. The calculated neutron separation energies exhibit the experimentally observed odd-even staggering effect. This

effect disappears when we neglect blocking. In calculating the odd-even mass differences, we show that the effective strength of the pairing force is inflated by 20 to 40% when the blocking effect is ignored.

The BCS trial wave function $|\Phi_0\rangle$, or the quasiparticle (q.p.) vacuum, can be obtained by performing the canonical Bogoljubov-Valatin (BV)⁴ transformation e^S on the single-particle (s.p.) vacuum state $|0\rangle$:

$$|\Phi_0\rangle = e^S |0\rangle, \quad (1)$$

with e^S defined in terms of the occupational u and v factors,

$$a_\alpha^\dagger = e^S c_\alpha^\dagger e^{-S} = u_\alpha c_\alpha^\dagger - (-)^{j_\alpha - m_\alpha} v_\alpha c_{\bar{\alpha}}, \quad (2)$$

where $u_\alpha^2 + v_\alpha^2 = 1$, $c_\alpha^\dagger$ (c_α) and $\bar{a}_\alpha^\dagger$ ($\bar{a}_\alpha$) are s.p. and q.p. creation (annihilation) operators, respec-

tively. The subscript a stands for all the quantum numbers of a s.p. state except the magnetic quantum number m_a , the Greek subscript α stands for (a, m_a) , and $\bar{\alpha} \equiv (a, -m_a)$. The parameters of the BV transformation are determined by the variational condition

$$\delta \langle \Phi_0 | H - \lambda \hat{N} | \Phi_0 \rangle = 0, \quad (3a)$$

and the constraint

$$\langle \Phi_0 | \hat{N} | \Phi_0 \rangle = N, \quad (3b)$$

where

$$H = \sum_{\alpha} \epsilon_{\alpha}^0 c_{\alpha}^{\dagger} c_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma}$$

is the usual Hamiltonian, λ is the chemical potential, $\hat{N} = \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$ is the number operator, N is the number of particles in the system, and ϵ_{α}^0 are the s.p. energies of the active particles due to the field of the inactive particles. It is convenient to define the following functions:

$$U_a(\Omega_a, \mu_a, \Delta_a) = \Omega_a [2(\epsilon_a^0 - \frac{1}{2}\mu_a)v_a^2 - \Delta_a u_a v_a], \quad (4)$$

$$M_a(\mu_a, \Delta_a) = 2(\epsilon_a^0 - \mu_a - \lambda)u_a v_a - \Delta_a(u_a^2 - v_a^2), \quad (5)$$

where $\mu_a = \sum_c F_{ac} \Omega_c v_c^2$ is the s.p. self-energy, $\Delta_a = \sum_c G_{ac} \Omega_c u_c v_c$ the pairing energy,⁵ and $\Omega_a = j_a + \frac{1}{2}$.

Equations (3) then lead to the familiar "gap" equations

$$M_a(\mu_a, \Delta_a) = 0, \quad 2 \sum_a \Omega_a v_a^2 = N. \quad (6)$$

For an odd system $|\Phi_0\rangle$ is not a suitable choice for the trial wave function. In this case there must be at least one particle which is not pair correlated with any other particle. A natural choice of the trial wave function when N is odd with one particle in the β orbital unpaired is^{2, 6}

$$|\Phi_{\beta}\rangle = e^S (c_{\beta}^{\dagger} |0\rangle) = a_{\beta}^{\dagger} |\Phi_0\rangle. \quad (7)$$

The variational condition and constraint on $|\Phi_{\beta}\rangle$ now leads to the new gap equations

$$M_a(\mu'_a, \Delta'_a) = 0, \quad 2 \sum_a \Omega'_a v_a^2 = N - 1, \quad (8)$$

where $\Omega'_a = \Omega_a - \delta_{ab}$, Δ'_a is defined in the same way as Δ except that Ω is replaced by Ω' , and

$$\mu'_a - \mu_a = \frac{1}{2}(u_b^2 - v_b^2) \left[F_{ab} - \frac{\delta_{ab}}{\Omega'_a} (G_{bb} - F_{bb}) \right]. \quad (9)$$

Note that for β such that $\Omega_b = 1$, $G_{bb} \equiv F_{bb}$, and we must have $u_b^2 = v_b^2 = \frac{1}{2}$, so that $\mu'_a - \mu_a \equiv 0$.

The ground-state energy for the state $|\Phi_0\rangle$ is

$$W = \langle \Phi_0 | H | \Phi_0 \rangle = \sum_a U_a(\Omega_a, \mu_a, \Delta_a), \quad (10)$$

and for $|\Phi_{\beta}\rangle$ is

$$W_{\beta} = \langle \Phi_{\beta} | H | \Phi_{\beta} \rangle = \sum_a U_a(\Omega'_a, \mu'_a, \Delta'_a) + \epsilon_b^0 - \frac{1}{2}\mu_b + \frac{1}{2}G_{bb}v_b^2. \quad (11)$$

As an application of Eqs. (8) and (11), we used the realistic nuclear free reaction matrix of Kahana, Lee, and Scott⁷ to calculate the residual binding energies of the ground and low excited states for the odd-mass Sn isotopes. The s.p. wave functions for ¹¹⁶Sn were generated by a spherical harmonic-oscillator potential well with $\hbar\omega = 8.3$ MeV. For other isotopes we assumed $\hbar\omega \propto A^{-1/3}$. The 12 active neutron orbitals and their s.p. energies ϵ^0 and $\epsilon^0 - \mu$ for ¹¹⁶Sn are given in Table I. The pairing correlations among the protons and between the protons and neutrons were neglected. However, half the sum of the s.p. energy shifts of the 22 protons in the (assumed) $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals, due to the active neutrons, i.e.,

$$\Delta W^{(p)} = \sum_{\text{proton}} \Omega_p \sum_{\gamma (\text{neutron})} F_{pc} \frac{1}{2} \langle \Phi | c_{\gamma}^{\dagger} c_{\gamma} | \Phi \rangle$$

where $|\Phi\rangle = |\Phi_0\rangle$ or $|\Phi_{\beta}\rangle$, for even or odd systems, is included in the residual binding energy.

Values for ϵ^0 were chosen such that the calculated low-energy spectra for the odd-mass isotopes best resembled the empirical ones obtained from data of other workers.⁸⁻¹⁰ These spectra are shown in Fig. 1. Also shown are the q.p. spectra calculated using (6), i.e., ignoring the blocking effect but using the same values for ϵ^0 . The differences between the blocked and unblocked results are significant. However, it does not follow from the comparison in Fig. 1 that the method including the blocking effect is the better approximation. By changing the values for ϵ^0 within reasonable limits the result obtained from the unblocked calculation can be substantially improved. A general trend of the theoretical result is that the $\frac{1}{2}^{+}$ q.p. state is not sufficiently bound. This has al-

TABLE I. Neutron s.p. energies ϵ^0 and $\epsilon^0 - \mu$ for ¹¹⁶Sn.

Orbit	ϵ_0 (MeV)	$\epsilon^0 - \mu$ (¹¹⁶ Sn)
$2p_{3/2}$	-5.5	-14.50
$1f_{5/2}$	-5.0	-13.48
$2p_{1/2}$	-4.0	-12.92
$1g_{9/2}$	-2.5	-10.87
$2d_{5/2}$	0.0	-7.52
$1g_{7/2}$	1.3	-6.13
$3s_{1/2}$	1.2	-5.91
$2d_{3/2}$	1.8	-5.04
$1h_{11/2}$	2.7	-4.53
$1h_{9/2}$	4.0	-2.14
$2f_{7/2}$	5.0	-1.25
$1i_{13/2}$	7.5	1.36

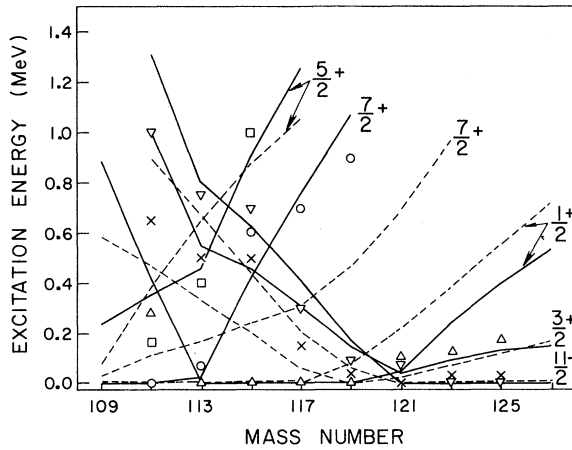


FIG. 1. Low-energy spectra of odd-mass Sn isotopes. The solid lines are calculated with the blocking effect included. The dashed lines are calculated ignoring the blocking effect. The symbols for the experimental data (Refs. 8-10) stand for: $\square$, $(\frac{5}{2}^+)$; $\circ$, $(\frac{7}{2}^+)$; Δ , $(\frac{1}{2}^+)$; $\times$, $(\frac{3}{2}^+)$; ∇ , $(\frac{11}{2}^-)$.

ready been observed by previous investigators,¹¹ who had neglected the blocking effect. The present calculation made some improvements on this. For example, whereas Kuo, Baranger, and Baranger¹¹ reported that the $\frac{1}{2}^+$ q.p. state in ^{125}Sn is too weakly bound by ~ 0.5 MeV, in the present calculation only ~ 0.2 is missing. The calculated levels above 0.5 MeV will in general be lowered somewhat if three-q.p. states are allowed to be mixed with the one-q.p. states. For example, the $\frac{3}{2}^+$ level for ^{111}Sn comes down by ~ 0.2 MeV in a particle-core intermediate-coupling calculation.¹²

The neutron separation energies, $S_N = -(W_N - W_{N-1})$, evaluated for the ground state are shown in Fig. 2. The odd-even effect is reproduced, al-

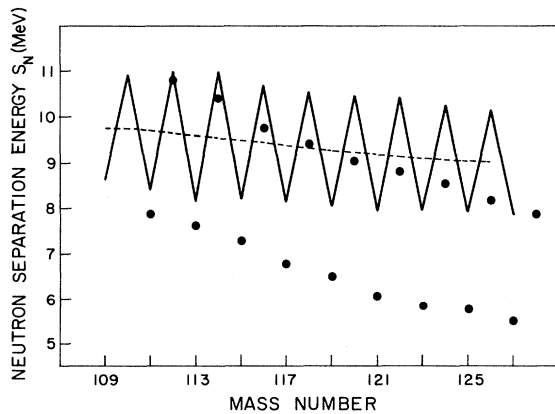


FIG. 2. Neutron separation energies $S_N = -(W_N - W_{N-1})$. The solid and the dashed lines are calculated with and without blocking, respectively. Data points are from J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. **67**, 32 (1965).

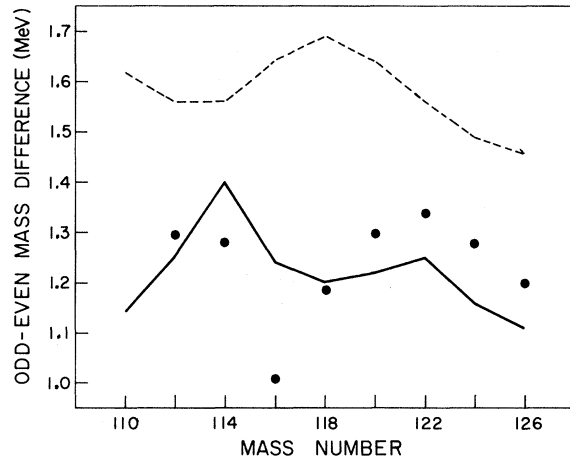


FIG. 3. Odd-even mass differences. The solid line connects points given by $D_N = \frac{1}{2}(S_N - S_{N-1})$. The dashed line connects $E_{\min}(N)$. Data points are from J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. **67**, 32 (1965).

though the over-all mass dependence is missing. The mass independence of the calculated separation energies in the Sn and Pb region was also reported by Baranger and co-workers¹³ using the Tabakin¹⁴ force. This result is probably due to the neglect of the proton-neutron symmetry energy, which cannot be treated unless the equilibrium of the nucleus as a whole is considered.¹⁵ The dashed line in Fig. 2 was obtained using (6) for even and odd systems alike, and does not show the odd-even effect, as expected.

In Fig. 3 we plot the "odd-even mass difference," $D_N = \frac{1}{2}(S_N - S_{N-1})$, for even N . The advantage of comparing these results with the empirical data is that the uncertainties in the absolute binding energies are eliminated. A simple argument¹⁶ leads to the first-order equality between D_N and the smallest q.p. energy, $E_a = [(\epsilon_a^0 - \mu_a - \lambda)^2 - \Delta_a^2]^{1/2}$, for the N system. The differences between the dashed and solid curves in Fig. 3 are the correction to this approximation. Significantly, the unblocked treatment inflates the odd-even mass difference by 20 to 40%. In other words, using the criterion $D_N \approx E_{\min}(N)$ to determine the strength of the pairing force would result in an underestimation of the strength. However, it has been shown by Nilsson,¹⁷ in two simple models both employing the constant pairing force, that the reduction of the odd-even mass difference by blocking is substantially canceled by corrections due to the changes in the widths of the particle number distributions in both the blocked and unblocked calculations. To what extent this cancellation still holds in our more realistic case is an interesting question. The dip of the empirical point at $A = 116$ is

another manifestation that the $\frac{1}{2}^+$ state acquires additional binding energy which the simple pairing theory cannot account for. Since in the s.p. shell model the $2d_{5/2}$ and $1g_{7/2}$ shells are closed at $A = 114$, it is tempting to interpret the anomaly at $A = 116$ as a shell effect. However, this can only be partially true, since the discrepancy between theory and experimental data for the $\frac{1}{2}^+$ level for $A > 119$ in Fig. 1 cannot be so explained. It is also unlikely that this is caused by the number-noncon-

serving property of the BCS theory, because the same effect persists over several mass numbers. The most probable cause may be due to a slight A dependence of the average s.p. field. This can become important because of the considerable change in the neutron number from $A = 109$ to $A = 125$. However, this conjecture should be investigated.

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$$V_{\alpha\beta\gamma\delta} \equiv \sum_j G_{abcd} c_{\alpha\beta}^j c_{\gamma\delta}^j \equiv \sum_{j'} F_{acdbj'} c_{\alpha\gamma}^{j'} s_{\beta} c_{\delta}^{j'} s_{\beta};$$

$$F_{ab} \equiv -\frac{1}{(\Omega_a \Omega_b)^{1/2}} F_{aabb0}; \quad G_{ab} \equiv -\frac{1}{2(\Omega_a \Omega_b)^{1/2}} G_{aabb0};$$

$$c_{\alpha\beta}^j \equiv \langle j_a m_a j_b m_b | j m_a + m_b \rangle; \quad s_{\alpha} \equiv (-)^{j_a - m_a}.$$

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