$\gamma_{pol} + N \to \pi^{\pm} + N$ AND THE HADRON WEAK CURRENT

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It is pointed out that a very direct way of testing current ideas of the QCD renormalization of the weak neutral interaction between hadrons is to measure the asymmetry in yield induced by left- and right-polarized photons in the reactions $\gamma_{pol} + N \to \pi^{\pm} + N$. Recent calculations predict the asymmetry to be in the range of 1.5 to 8.0×10^{-7}.

Essentially all experiments that have provided us with definite information on hadron weak neutral currents involve semi-leptonic reactions. Recent model-independent analyses [1] of neutrino and antineutrino scattering data have shown that as far as u and d quarks are concerned, the $SU(2) \otimes U(1)$ model of Weinberg and Salam (WS) [2] is consistent with all such data provided the mixing angle θ_{WS} satisfies $\sin^2\theta_{WS} \approx 0.30$. Moreover, earlier fears that the atomic bismuth experiments [3] may refute the WS description of the weak, neutral electron–nucleon interaction have been mostly allayed by the recent measurements [4] by the SLAC–Yale group of parity violation in $e + d \to e + d$ and $e + p \to e + p$, the result of which is explained by the WS model with $\sin^2\theta_{WS} = 0.20 \pm 0.03$.

In contrast, other than the $\Delta S = \Delta Q$ selection rule leading to the Glashow–Iliopoulos–Maiani (GIM) [5] scheme for strangeness-conserving currents, there is no experimental data which can be used to determine directly the s, c, ... quark contents of the hadron neutral current. One class of reactions which could tell us something about these contents involves the study of parity-nonconserving effects induced by weak interaction in the nucleus. There has been a vigorous program of such studies experimentally [6] and theoretically [7] in the past several years.

A very important difference between leptonic or semileptonic weak interactions and the nonleptonic weak interaction is that the latter suffers short-distance renormalizations by the strong interaction. Thus it is believed that these effects are responsible for the enhancement of the (strangeness changing) $\Delta I = \frac{1}{2}$ term and the suppression of the $\Delta I = \frac{3}{2}$ term in the hyperon decays [8]. Recent QCD calculations [9] of the gluon correction to strangeness-conserving but parity-violating hadronic weak interaction have predicted a very large increase in the weak $NN\pi^{\pm}$ amplitudes as compared to those expected from the unrenormalized Cabibbo weak interaction [10]. Here we point out that the parity-violating $NN\pi$ coupling constant f_{π}^{W} can be directly measured from the asymmetry in $\gamma_{pol} + N \to \pi^{\pm} + N$ *near threshold* with circularly polarized photons.

The most important aspect of the asymmetry near threshold (up to ~20 MeV above threshold) is that it is sensitive only to the parity violating $NN\pi$ amplitude. At energies significantly above threshold, the parity-violating vertices $N\Delta\pi$, $NN\rho$ and $NN\omega$ also come into play, and the physics becomes more entangled. The major disadvantage of studying the asymmetry near threshold is experimental: low count rates and possibly the difficulty in detecting low-energy charged pions.

To calculate the parity-violation effect in $\gamma_{pol} + N \to \pi^{\pm} + N$ we use the chiral lagrangian of Peccei [11] to describe the normal π photoproduction. For the weak production we use an effective lagrangian [12] with the scalar $NN\pi$ coupling (we ignore the weak, parity-conserving pseudoscalar term):

$$\mathcal{L}^{W}_{NN\pi} = 2^{-1/2} f^{W} \bar{N} (\vec{\tau} \times \vec{\pi})_3 N \,, \tag{1}$$

$$f^{W}_{\pi} = G_{W} m^2 a(n^0_-) \,, \tag{2}$$

where $G_W = 1.0 \times 10^{-5}\, M^{-2}$ is the universal weak coupling constant, M is the nucleon mass, m is the pion mass and $a(n^0_-)$ is the dimensionless [12], weak, s-wave $n \to p + \pi^-$ amplitude. The twelve amplitudes for the normal π production are calculated from the Feynman diagrams in fig. 1 and are given in ref. [11]. We calculate the corresponding amplitudes for parity violating π production from the diagrams in fig. 2. In this case the gauge term corresponding to fig. 1d does not appear because $\mathcal{L}^{W}_{NN\pi}$ has a scalar coupling as opposed to the pseudovector coupling used by Peccei [11] for the strong lagrangian. We obtain [‡1]

$$a^{W}_{i}(\gamma N \to \pi^{\pm} N) = \sum_{F=A,B,C,D} \; \sum_{\circledcirc=+,-,0} \bar{N}_1 F^{\circledcirc} M_F O^{\circledcirc}_i(\tau) N_2 \,, \tag{3}$$

where i is the isospin index of the pion; M_F are the dimensionless scalar counterparts of the four pseudoscalar operators, $M^5_F = M_F \gamma_5$, of Chew–Goldberger–Low–Nambu [13];

[‡1] The four-momenta P, q, k and the invariant masses s, t, u are those defined in refs. [11] and [13]. Specifically $k = (\boldsymbol{k}, \omega)$ is the four-momentum of the photon and ϵ its polarization. We use the convention $P \cdot k \equiv \boldsymbol{P} \cdot \boldsymbol{k} - P_0 k_0$ and $\not{k} = \gamma \cdot k$, where γ_μ are the Dirac matrices.

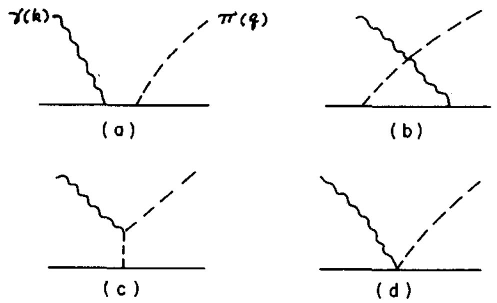

Fig. 1. The four lowest-order Feynman diagrams used by Peccei [11] to compute the $\gamma N \to \pi N$ amplitudes near threshold. Diagram (d) represents the gauge term coming from the pseudovector $NN\pi$ strong coupling used in ref. [11]. Solid lines represent nucleons, wavy lines represent photons, and dashed lines represent gluons.

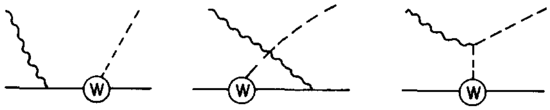

Fig. 2. The three diagrams used in this paper to compute the parity violating $NN\pi$ amplitude, eq. (3).

$$M_A = \not{\epsilon}\not{k}/m \,,$$

$$M_B = 2(P\cdot k\, q\cdot\epsilon - P\cdot\epsilon\, q\cdot k)/(M^2 m) \,,$$

$$M_C = (q\cdot\epsilon\, \not{k} - q\cdot k\, \not{\epsilon})/(Mm) \,;$$

$$M_D = 2(P\cdot\epsilon\, \not{k} - P\cdot k\, \not{\epsilon} + M\not{\epsilon}\not{k})/(Mm) \,;$$

$$O^{+}_i = \delta_{i3} \,, \quad O^{-}_i = \tfrac{1}{2}[\tau_i, \tau_3] \,, \quad O^{0}_i = (1-\delta_{i3})\tau_i \,.$$

These operators are manifestly gauge invariant. The twelve invariant functions $F^{\circledcirc}$ are

$$A^{+} = B^{+} = C^{+} = D^{+} = 0 \,, \tag{4a}$$

$$A^{-} = -\frac{t-m^2}{M^2} B^{-} = \frac{2}{\kappa_V} C^{0} = \frac{2}{\kappa_S} D^{-}$$

$$= -\tfrac{1}{2} e f^{W}_{\pi} m \left(\frac{1}{s-M^2} + \frac{1}{u-M^2} \right) , \tag{4b}$$

$$A^{0} = -\frac{t-m^2}{4M^2} B^{0} = \frac{2}{\kappa_S} C^{-} = \frac{2}{\kappa_V} D^{0}$$

$$= -\tfrac{1}{2} e f^{W}_{\pi} m \left(\frac{1}{s-M^2} - \frac{1}{u-M^2} \right) , \tag{4c}$$

where s, t and u are the invariant masses squared in the γN_2, $N_1 N_2$ and $N_2 \pi$ channels, respectively. Eq. (4a) means there is no π^0 production; it is a direct consequence of the structure of the CP-conserving $\mathcal{L}^{W}_{NN\pi}$ given in eq. (1).

Near threshold in the laboratory frame with the z-axis along the incident photon direction, the total production-amplitude squared, for circular photon polarization $\mu = \pm 1$, is

$$|a(\gamma N \to \pi^{\pm} N)|^2_{\mu} = e^2 f^2_{\pi} (1-2\rho)^2$$
$$\times \{ [(1 \pm \rho)^2 \langle \sigma_{-\mu} \rangle^2 + \tfrac{1}{2} (q \sin\theta/\omega)^2 \langle \sigma_3 \rangle^2]_a$$
$$+ 2 m f^{W}_{\pi}/(\omega f_{\pi}) ([\mu\rho (1 \pm \rho)(1 + \kappa_V) \langle \sigma_{-\mu} \rangle^2]_b$$
$$- [(q \sin\theta/\omega)^2 \langle \sigma_3 \rangle]_c) \} \,, \tag{5}$$

where θ is the pion scattering angle, q is the pion momentum, ω is the photon energy, $\rho = \omega/2M$, and $\langle\sigma_\alpha\rangle$ is the spin matrix element for the nucleon states.

We have ignored the contributions from the Δ(1232 MeV) resonance which is known to be $\lesssim 10\%$ up to 20 MeV above threshold (in the laboratory frame). Also ignored in the calculations are the ρ- and ω-pole terms represented by fig. 3a. Near threshold these terms are of the order $(m^2/m^2_{\rho,\omega})$. The contribution from fig. 3b, also ignored, could be large due to the strong ρ–γ coupling and the possibly large, parity-violating NNρ amplitude, only if we assumed the ρ–γ amplitude to be proportional to m^2_ρ even for the light-like ($k^2 = 0$) virtual ρ meson. Although the continuation of the ρ–γ conversion from the time-like to light-like region is not well known, we shall assume that for $|\boldsymbol{k}|^2 = k_0^2 \ll m^2_\rho$, the coupling is weak, and fig. 3b is also of order $k_0^2/m^2_\rho \approx m^2/m^2_\rho$ and can be ignored.

In eq. (5) the term designated $[\ ]_a$ is from the regular NNπ amplitude. Terms $[\ ]_b$ and $[\ ]_c$ arise from the interference between the strong and weak interactions with opposite parities. Note that $[\ ]_b$ changes sign with the circular polarization of the photon and is isotropic and q-independent. In contrast, $[\ ]_c$ changes

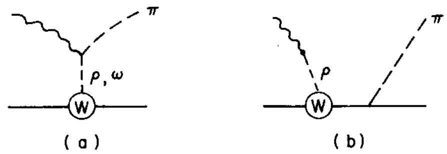

Fig. 3. (a) The ρ- and ω-pole terms involving the weak NNρ and NNω vertices and (electromagnetic) $\gamma\pi\rho$ and $\gamma\pi\omega$ couplings. (b) the ρ-pole term involving the direct $\gamma\rho$ conversion. These diagrams are ignored in our calculation of weak π production near threshold.

sign with the longitudinal polarization of the target nucleon and is proportional to $q^2 \sin^2\theta$. Thus near threshold $|[\ ]_b| \gg |[\ ]_c|$.

From eq. (5), the asymmetry in the cross section for circularly polarized photon is, to leading order

$$A_\gamma \equiv \frac{\sigma_{\gamma\uparrow} - \sigma_{\gamma\downarrow}}{\sigma_{\gamma\uparrow} + \sigma_{\gamma\downarrow}} = \frac{f_\pi^{\mathrm{W}}}{f_\pi} \frac{m}{M} (1 + \kappa_{\mathrm{V}}) . \qquad (6)$$

The strong coupling constant is $f_\pi = 1.0$ (i.e. $f_\pi^2/4\pi = 0.08$) and $1 + \kappa_{\mathrm{V}} = 4.7$. Therefore from eqs. (2) and (6)

$$A_\gamma = 1.6\, a(n^0_-) \times 10^{-7} . \qquad (7)$$

Following the prevalent convention we write

$$a(n^0_-) = R a(n^0_-)_{\mathrm{c}} , \qquad (8)$$

where

$$a(n^0_-)_{\mathrm{c}} = \sqrt{2/3} \tan\theta_{\mathrm{c}}\ [2a(\Theta^0_-) - a(\Lambda^-_-)] = 0.17$$

is the amplitude [12] obtained from the SU(3) sum rule [14–16] in the Cabibbo model and R is the "enhancement factor" due to contributions from neutral currents and renormalization effects; θ_{c} is the Cabibbo angle with $\sin^2\theta_{\mathrm{c}} = 0.053$. The evaluation of R depends on the structure of the neutral currents, the renormalization procedure and the quark structure of the hadrons. Recent calculations [9] of R, based on the Weinberg–Salam model for the neutral current, have yielded the results shown in table 1. One of the conclusions reached in these studies is that the dominant contribution to $a(n^0_-)$ yields a sign for f_π^{W} which is the same as that for f_π. From eq. (6), this implies that the asymmetry under study has a positive sign. Based on the results shown in table 1, the asymmetry

Table 1
Result for $R = a(n^0_-)/a(n^0_-)_{\mathrm{c}}$, f_π^{W} and A_γ from recent QCD calculations.

R	$f_\pi^{\mathrm{W}} \times 10^7$	$A_\gamma \times 10^7$	Authors
~25	~9.3 a)	~6.8	Körner et al. [8]
16~29	6.0~11	4.4~7.9	Buccella et al. [8]
5~13	1.9~4.8 b)	1.4~3.5	Guberina et al. [8]
~12	~4.4	~3.3	Desplanques et al. [8]

a) Körner et al. used a phase convention such that both $a(n^0_-)_{\mathrm{c}}$ and f_π^{W} are negative. We have adapted their result to our phase convention.

b) Guberina et al. used the approximation $G_{\mathrm{W}} m^2 a(n^0_-)_{\mathrm{c}} \approx 5 \times 10^{-8}$. We use $G_{\mathrm{W}} m^2 a(n^0_-)_{\mathrm{c}} = 3.8 \times 10^{-8}$ and have scaled their results for f_π^{W} downward accordingly.

is expected to be in the range of 1.5 to 8.0×10^{-7}.

It is not clear how feasible it would be to measure the asymmetry under discussion. There does not seem to be any technical barrier in the generation of an intense, circularly polarized photon beam, such as from the bremsstrahlung of linearly polarized electrons [17]. The detection of fairly low-energy pions ($\lesssim$20 MeV) to a sensitivity of $< 10^{-6}$ would be extremely difficult, however. Earlier we have already mentioned that $\gamma_{\text{pol}} + \text{N} \to \pi^{\pm} + \text{N}$ is of special interest near threshold because it is a direct measure of the parity-violating $\text{NN}\pi$ coupling alone. At higher energies it might be easier to detect the pions but the physics also would become more entangled due to the growth in importance of the Δ and heavier meson-pole terms (see fig. 3). Nevertheless, asymmetry in this energy region will eventually be useful in providing constraints on the weak NN–meson and NΔ–meson coupling which at the moment have large theoretical uncertainties.

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Note added in proof: Recently a paper concerning weak interaction and pion photoproduction by R.M. Woloshyn has appeared in Can. J. Phys. 57 (1979) 809. In this paper experimental aspects of low energy pion photoproduction are discussed in detail.

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