ELECTROEXCITATION AND THE DETERMINATION OF THE K-BAND STRUCTURE IN ^{24}Mg ☆

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The high-resolution electron scattering facility at the MIT-Bates accelerator was used to resolve the 4_1^+ 4.12 MeV and 2_2^+ 4.24 MeV levels in ^{24}Mg. The respective E2 and E4 Coulomb form factors were measured and compared to form factors calculated theoretically; the 4_1^+ form factor exhibits a momentum-transfer dependence which strongly suggests that K is a good quantum number in ^{24}Mg.

In this letter we report the results of a recent (e, e′) experiment on ^{24}Mg and, by comparison of the strengths and shapes of the form factors with those from a projected Hartree–Fock (PHF) calculation, surmise that the lowest $J^{\pi} = 2^+$ and 4^+ states must belong to almost pure $K = 0$ and $K = 2$ bands. This conclusion is possible because we have, for the first time, managed to resolve the excitations to the 4_1^+ 4.12 MeV and the 2_2^+ 4.24 MeV levels. A further comparison with a renormalised spherical shell model (SM) calculation yields insight into the use of effective operators. Hence even though the shell model appears to successfully account for $B(\mathrm{E}\lambda)$ values of low-lying 2^+ and 4^+ states in ^{24}Mg, the new electron scattering data show that this description is very far from complete.

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The present (e, e′) experiments were performed with the high-resolution electron scattering facility at the MIT-Bates accelerator [1]. A portion of a typical inelastically-scattered electron spectrum is shown in fig. 1, taken at a spectrometer angle of 90.0° and an incident electron energy of 218.1 MeV. Here the 4.12 and 4.24 MeV levels are clearly resolved. A target of thickness 25.2 ± 0.1 mg/cm^2, area 4.5 cm × 4.0 cm, and isotopic purity of 99.4% ^{24}Mg was used. The measurements were made relative to the observed elastic peak, and to the inelastic peaks of the 2_1^+ state at 1.37 MeV and the 4_2^+ state at 6.01 MeV excitation in ^{24}Mg, the Coulomb form factors of which have been previously measured [2–4]. Normalization was also made relative to the elastic peak of ^{12}C, observed with a graphite target.

The form factor for inelastic electron scattering to an isolated level is given in plane-wave Born approximation (PWBA) by [5]

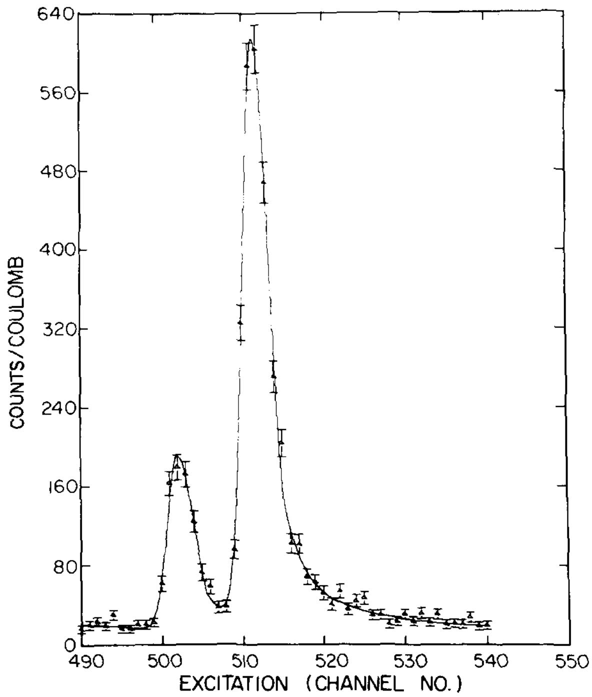

Fig. 1. A scattered electron spectrum from ^{24}Mg, taken at 90.0° and 218.1 MeV, showing the resolved 4_1^+ and 2_2^+ levels.

$$F_{C\lambda}^2(q) + [\tfrac{1}{2} + \tan^2(\theta/2)]\, F_{T\lambda}^2(q) = (d\sigma/d\Omega)/Z^2 R\sigma_M, \tag{1}$$

where $d\sigma/d\Omega$ is the measured cross section with radiative correction applied,

$$\sigma_M = \frac{\alpha^2 \cos^2(\theta/2)}{4E_e^2 \sin^4(\theta/2)}, \quad R = \left[1 + \left(\frac{2E_e}{M_T}\right)\sin^2(\theta/2)\right]^{-1}, \tag{2}$$

E_e is the incident electron energy and θ is the electron scattering angle; $F_{C\lambda}^2(q)$ is the Coulomb form factor and $F_{T\lambda}^2(q)$ is the transverse form factor which may be electric or magnetic. The Coulomb form factors for the 2_2^+ 4.24 MeV and 4_1^+ 4.12 MeV levels (shown by the black squares) are plotted respectively in figs. 2 versus the effective momentum transfer, i.e. the momentum transfer renormalized to remove the effects of distortion. The data points marked by open circles at low momentum transfer were taken from Johnston and Drake [4]. For each level the measured differential cross section is compared to that calculated with the distorted-wave code DUELS using a Tassie model for the transition charge density [6]. In these calculations the parameters c and t of a Fermi charge density, $\rho(r) = \rho_0[1 + \exp(r - c)4.4/t]^{-1}$, were varied to fit the data; see table 1.

Table 1
Parameters for Fermi transition charge densities to the 2^+ and 4^+ $K = 0$ and 2 states in ^{24}Mg.

J^π	c(fm)	t(fm)	$B(E\lambda\uparrow)$
2_1^+	2.77	2.35	$453 \pm 35\ e^2\text{fm}^4$
2_2^+	2.77	2.35	$27.4 \pm 3.0\ e^2\text{fm}^4$
4_1^+	3.625	1.85	$(2.0 \pm 0.3) \times 10^3\ e^2\text{fm}^8$
4_2^+	2.725	1.91	$(4.3 \pm 0.6) \times 10^4\ e^2\text{fm}^8$

We first put the experimental results for the 2_2^+ and 4_1^+ states of ^{24}Mg into the perspective suggested by existing data on the 2_1^+ and 4_2^+ states. The 2_2^+ form factor has a maximum value which is smaller than that of the 2_1^+ by a factor of 5.8×10^{-2}, yet the shapes are similar. Such is not the case for the 4_1^+ and 4_2^+ form factors. The 4_1^+ form factor is not only smaller in magnitude than that of the 4_2^+ by a factor of 4.6×10^{-2}, but it is very different in shape, the diffraction minimum being at 2.0 fm^{-1} rather than 2.5 fm^{-1}, which implies a very large transition radius.

Shell model calculations limited to the 0d–1s shell, but complete within this space, are able to systematically account for many aspects of the structure of low-lying states [7]. Observed state-to-state variations in E2 strength are generally well predicted if the nucleons of the model space ($A - 16$ in number) carry added charges of $\approx 0.35e$ [8]. In this context the effective charges represent an average renormalization which corrects for the restriction to the single shell. These shell-model wave functions were used, together with the assumption of mass-state-q independent effective charges to calculate form factors of the relevant states of ^{24}Mg in the PWBA. Harmonic oscillator wave functions were used, with the oscillator constant chosen to fit the ground-state charge distribution. The form factors for the strong transitions to 2_1^+ and 4_2^+ states are well accounted for in both magnitude and shape by these calculations, as are similar data from ^{20}Ne and ^{28}Si [9].

Turning to the data of interest here, we note that the calculated form factors for the 2_2^+ state are larger in magnitude than the experimental result. By reducing

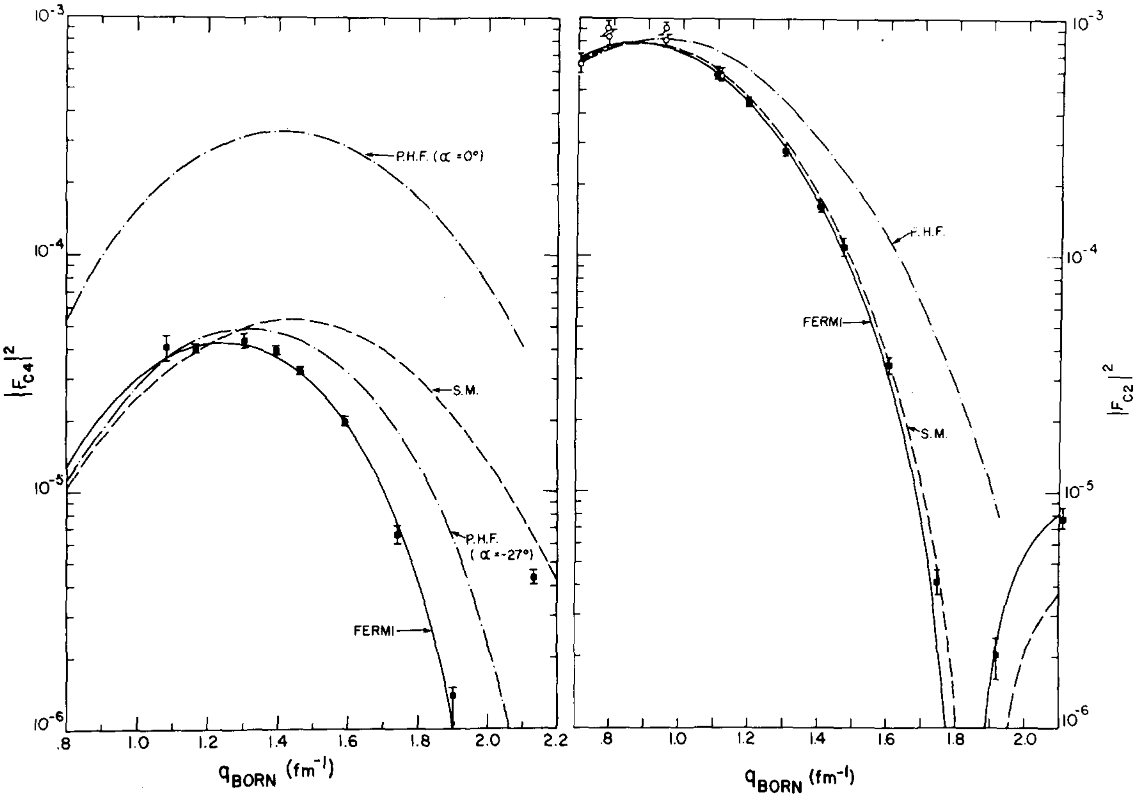

Fig. 2. The form factor for the 4_1^+ 4.12 MeV level of ^{24}Mg is on the left, and for the 2_2^+ 4.24 MeV level on the right. The Fermi-density form factors are shown as a solid line, shell-model form factors as a dashed line and the PHF form factors as a dash–dot line.

this theoretical form factor by 0.615 and comparing the data, it can be seen in fig. 2 that the observed shape is well reproduced. The magnitude of the maximum of the calculated 4_1^+ form factor is approximately that measured, however, the calculated shape is dramatically different from that observed. Of course this calculated shape is fixed by our model assumptions since only the d–d matrix elements contribute to it. Thus all calculated 4^+ form factors are alike and, as it happens, will match the 4_2^+ shape. The experimental 4_1^+ shape is therefore completely anomalous. Hence, even though the 0d–1s shell model appears to successfully account for the B(E4) values of the 4^+ states in ^{24}Mg, the new electron scattering data show that this description is very far from complete.

We now turn to a comparison of the (e, e′) data with the results of PHF calculations [10]. These calculations were carried out within a 5 $\hbar\omega$ space using the Saunier–Pearson interaction [11]. In this large space *no* renormalization of the nucleon charge is required. It was found that the 2_1^+ and 2_2^+ states had almost pure $K = 0$ and $K = 2$ band structures, respectively. The 4_1^+ state, whilst having a major $K = 0$ component, had also a 14% $K = 2$ admixture; the 4_2^+ state had the approximate orthogonal combination. In the calculation of form factors it was found that that for 2_1^+ is in good agreement with experiment up to the first diffraction minimum; that for 2_2^+(cf. fig. 2) agrees with experiment up to and slightly beyond the peak of the form factor, but exhibits a discrepancy at $q \approx 1.7$ fm^{-1} equivalent to an underestimation of 7% in the size parameter for the transition density. For 4_1^+, it is seen in fig. 2 that the PHF calculation overestimates the strength of the form factor by an order of magnitude and predicts a radically wrong shape with a diffraction minimum at ≈2.5 fm^{-1} rather than 2.0 fm^{-1}. An analysis [10] of earlier published data has already shown that in ^{24}Mg most of the E4 strength from the ground state is concentrated in the transition to the 4_2^+ state; in contrast the PHF using the Saunier–Pearson interaction predicts the strength to be about evenly distributed between the 4_1^+ and 4_2^+ states. In an attempt to understand this discrepancy, we have observed that the Saunier–Pearson interaction (through the PHF procedure) does bring the 2_1^+, 2_2^+, 4_1^+ and 4_2^+ states low in energy, but does not necessarily sort them out correctly. That an inadequacy does exist is evident in the failure of the calculation to yield the observed $2_1^+-2_2^+$ and $4_1^+-4_2^+$ energy differences. We have thus recalculated the form factors with the states $|4_{\mathrm{I}}^+\rangle = \cos\alpha\,|4_1^+\rangle + \sin\alpha\,|4_2^+\rangle$ and its orthogonal combination. The angle α is determined from a better fit to the (e, e′) data. We find $\alpha = -26.7°$ reduces the strength of the form factor for 4_{I}^+ by an order of magnitude and radically reduces (see fig. 2) the diffraction minimum from $\approx 2.5\ \mathrm{fm}^{-1}$ almost down to that observed. An angle $\alpha = -26.7°$ increases the strength for 4_{II}^+ by about a factor of two with little effect on the form factor shape (not shown, but see ref. [10], fig. 10). Both of these changes yield much better agreement with experiment, with the remaining discrepancy at $q \approx 1.7\ \mathrm{fm}^{-1}$ for 4_{I}^+ similar to that for 2_2^+. The calculated $B(\mathrm{E4}, 0_1^+ \to 4_{\mathrm{I}}^+)$ is $2.43 \times 10^3\ e^2\mathrm{fm}^8$. When the states $|4_{\mathrm{I}}^+\rangle$ and $|4_{\mathrm{II}}^+\rangle$ are reexpressed in terms of the K-bands (cf. ref. [10], table 4), we now find that $|4_{\mathrm{I}}^+\rangle$ is almost pure $K = 0$ and $|4_{\mathrm{II}}^+\rangle$ is almost pure $K = 2$ to within the non-orthogonality restrictions of states with different K.

The ability of the PHF wavefunctions to predict the strength of the $J^\pi = 2^+$ and 4^+ form factors with pure K-bands implies that the gross properties of the transition densities are being adequately described. The 4_1^+ form factor is extremely sensitive to K-band mixing, and the ability of the PHF calculation to predict its shape is attributed to the inclusion of higher shells in the PHF wave function, this being the first example of a form factor which exhibits a shape so strongly sensitive to the inclusion of the higher shells. The similar remaining disagreement between the calculated and measured 2_2^+ and 4_1^+ form factors at larger momentum transfers implies errors in the PHF transition densities at smaller radii. It remains to be seen whether these remaining discrepancies can be remedied by a change of interaction in the PHF framework, by enhancing the cluster structure of the states involved, or by some effect as yet unrecognised. Clearly part of the challenge of these new (e, e′) data is to find the correct physical interpretation of the remaining form factor discrepancies.

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