A COMMENT ON BAND SPLITTING IN ^{24}Mg

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Received 25 January 1975

The band splitting in ^{24}Mg arising from an unrestricted Hartree-Fock calculation is shown to be sensitive to an effective spin orbit interaction. Speculation is given on the origin of this spin-orbit effect.

The failure of standard effective residual interactions (e.g., those of Kuo and Brown and Kuo [1]) to reproduce the band splitting in ^{24}Mg (and other light nuclei) has been the subject of several recent communications [2–4]. That the correct band splitting can be reproduced within the framework of the spherical shell model has been demonstrated by McGrory and Wildenthal [2] using fitted two-body matrix elements. The characterisation of the change between the fitted and the standard matrix elements has not been easy, however. The suggestion of Feldmeier et al. [3] that it is due to the strengthening of the ALS part of the effective residual interaction seems to be in some doubt [4].

Of interest is the origin of the changes that have to be made. Do the changes arise for example from the renormalisation (core-polarisation etc.) correction terms that must be put into the effective residual interaction or is it the bare G-matrix which is in error? In this regard we note the results of the unrestricted deformed Hartree-Fock calculation of Cusson and Lee [5] for ^{24}Mg. Here an attempt was made to reproduce at least the collective features of the low energy spectrum within the wave functions of the projected (or variation-after-projection) "unrestricted†" deformed Hartree-Fock state (PHF or VPHF) – in contrast to the spherical shell model where much of the collective contributions come from the renormalisation in the effective operators. The surprising aspect of the Hartree-Fock results was the failure also to reproduce the lowest $K = 0$ and $K = 2$ band splitting even though the properties within the band could be reproduced with "bare" operators.

The $K = 2$ band in the Hartree-Fock calculation appears as a result of the γ-deformation of the Hartree-Fock (intrinsic) state. Thus one suspects that the band splitting will be sensitive to the magnitude of the γ-deformation. However, the variation-after-projection calculation of Cusson and Lee [5], allowing for changes in the γ-deformation between the $K = 0$ and $K = 2$ bands, also failed to yield a stable solution with a band splitting comparable with experiment. Thus one is tempted to seek changes in the interaction itself that will induce changes in the γ-deformation and hence changes in the band splitting.

It is the purpose of this note to point out the extreme sensitivity of γ-deformation and band splitting to the effective spin-orbit interaction in the *unrestricted* Hartree-Fock calculation. To demonstrate this we consider the Hamiltonian

$$H = H_{\mathrm{S.P.}} + x \sum_i s_i l_i \tag{1}$$

where $H_{\mathrm{S.P.}}$ is the Hamiltonian with the Saunier-Pearson interaction [6] as used by Cusson and Lee [5]. The Hamiltonian $H_{\mathrm{S.P.}}$ includes spin-dependent (vector and tensor) operators and thus itself leads to some spin-orbit splitting in the Hartree-Fock field [7].

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† Unrestricted means that Hartree-Fock orbits are not restricted to a single major spherical shell.

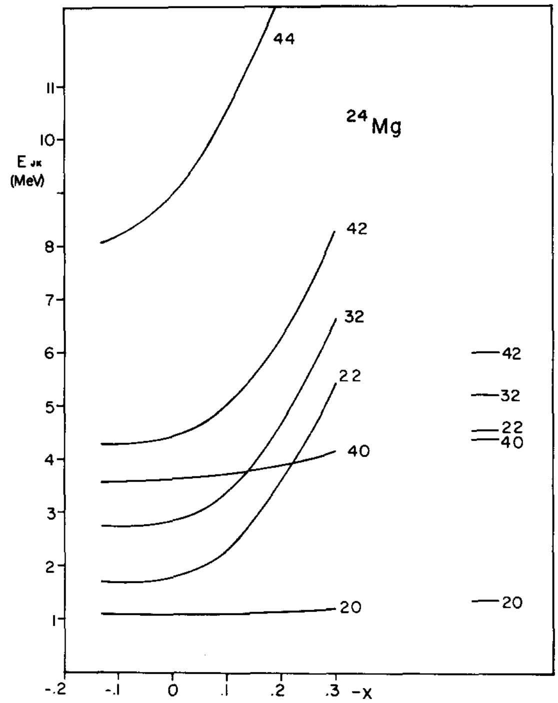

Fig. 1. Variation in the energy of the lowest bands in ^{24}Mg as a function of the strength of an addition spin orbit force to the Saunier Pearson [6] potential. Each state is defined by the *JK* labels and the experimental spectrum is shown on the right.

The additional spin-orbit term in eq. (1) thus either enhances or inhibits the spin-orbit splitting as $x < 0$ or $x > 0$. In fig. 1 is shown the variation with respect to x of the projected spectrum for ^{24}Mg relative to the $J = 0$ ground state as calculated by the computer program EVALIN [5]. That the increase in the band splitting is correlated with the decrease in γ-deformation is shown in fig. 2. With these results one must be sympathetic to the claims of Feldmeier et al. [3] that a change in a spin-dependent potential will lead to a larger band splitting. The physical origin, however, seems to be in the change of γ-deformation in the intrinsic state which can, as shown, be induced by changes in the spin-dependence in the interaction – this spin-dependent change being simulated here by the one-body spin orbit potential. The question obviously arises of possible origins for this spin-dependent change. We can suggest two.

Extensive perturbation calculations [8] in ground state energies of closed shell nuclei have shown the

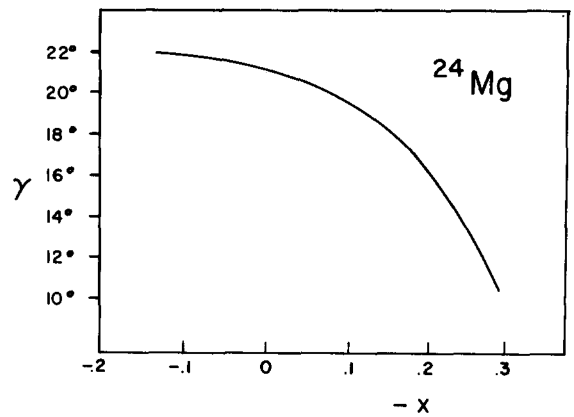

Fig. 2. Variation of the γ-deformation of the ^{24}Mg intrinsic state as a function of the additional spin orbit force.

importance of contributions from the tensor force arising from shells differing in energy from the occupied shells by $\Delta\hbar\omega \sim 6$ to $10\hbar\omega$. In the deformed Hartree-Fock calculations only contributions from neighbouring shells ($\Delta\hbar\omega \sim 2\hbar\omega$) could be taken into account because of limitations in computer space. Thus one must question whether the full collective effects of the tensor force have been adequately included in the deformed Hartree-Fock calculation with a "bare" interaction or whether here too one must consider some renormalisation. Since it is missing effects of the tensor force that one would like to simulate in the Hartree-Fock calculations then it is not unreasonable that these might be simulated by an effective (additional) spin orbit force.

A second possibility concerns the structure of single particle orbitals. The effective energy ϵ_j of an orbital with definite angular momentum j can be deduced from the deformed Hartree-Fock solution using the formula

$$\epsilon_j = \sum_{\nu} |U_\nu^j|^2 \tilde{\epsilon}_\nu/(2j+1)$$

where U_ν^j is the amplitude of the contribution of the j-orbit in the deformed Hartree-Fock orbit ν with energy $\tilde{\epsilon}_\nu$. Using such a formula one finds that the $d_{3/2}$ orbit in ^{24}Mg has much less binding than the $d_{5/2}$ – i.e., it is closer to becoming unbound. Thus in principle the $d_{3/2}$ wave function should have a different radial wave function than the $d_{5/2}$. This change is not entirely described in present Hartree-Fock calculations because of limitation in computer storage. The change, however, effectively builds a spin-dependence in the two-body matrix elements associated with the $d_{3/2}$ and $d_{5/2}$ orbits.

One of us (P.A.) wishes to thank G.A. Bartholomew and G.E. Lee-Whiting for the hospitality of the Physics Division at Chalk River during the summer of 1974. The assistance of D. McPherson and the computer staff is gratefully acknowledged.

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