ELECTRON SCATTERING FORM FACTORS FOR ^{12}C IN THE HARTREE–FOCK APPROXIMATION

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Elastic and inelastic electron scattering form factors for the ground (0_1^+) and first 2^+ and 4^+ states are calculated in the unrestricted, projected, Hartree–Fock approximation and compared with experimental data. A practical method, applicable to any type of wavefunction, is given for the calculation of the center-of-mass correction.to the form factor.

The study of electron scattering form ^{12}C has been extensive both experimentally [1–3] and theoretically [4–6]. Here we report calculations of form factors for the elastic and inelastic scattering to the ground (0_1^+), first 2^+ (4.44 MeV) and first 4^+ (14.1 MeV) states using wavefunctions obtained in the variational, self-consistent, projected, Hartree–Fock approximation (HFA). The results are insensitive to the one adjustable parameter appearing in the calculations. The agreement between theory and experiment is very good for the square of the momentum-transfer, q^2, up to $\sim 4F^{-2}$, but begins to deteriorate for larger values of q^2. We also point out a practical and straightforward method, applicable to any type of wavefunction, for calculating the center-of-mass (CM) correction to the form factors.

We calculate only the Coulomb form factor in the Born approximation. This will be sufficiently accurate except at diffraction minima [7]. In the Born approximation the Coulomb form factor of multipolarity λ is given by [4]

$$|F_{\lambda;J_\mathrm{i}\to J_\mathrm{f}}(q^2)|^2 = (4\pi/Z^2) \times \Big|\sum_k^Z \langle J_\mathrm{f}\| j_\lambda(qr_k) Y_\lambda(\Omega_k)\| J_\mathrm{i}\rangle\Big|^2 F_\mathrm{pr}^2(q^2) f_\mathrm{CM}^2(q^2). \quad (1)$$

Here the initial and final states are represented by projected HF wavefunctions [8]. These wavefunctions are expanded in a five major-shell "large" basis. F_pr is the electric form factor of the proton. Here it is represented by Stanford three-pole fit [9]. f_CM is the CM correction. This factor has most often been assumed to have the Tassie–Barker (TB) form [10], $f_\mathrm{CM}(q^2) = \exp(\frac{1}{4}A^{-1}b^2q^2)$, where A is the mass number of the target and b the oscillator length parameter. The TB form is correct only if (*i*) the wavefunction is a product of wavefunctions describing the internal motion and the CM motion and (*ii*) the CM is in the lowest harmonic oscillator state. In fact we have already assumed (*i*) in eq. (1) as the CM correction is factored out. In the following we describe a method to calculate f_CM in cases when (*ii*) does not necessarily hold. This method should be useful for any type of wavefunction consistent with (*i*).

Since f_CM is a function of q^2 only and $f_\mathrm{CM}(0) = 1$, we have

$$f_\mathrm{CM}(q^2) = 1 + \alpha_2 q^2 + \alpha_4 q^4 + \ldots . \quad (2)$$

To determine the coefficients α_2, α_4 etc. we need only consider the elastic Coulomb form factor $F_0(q^2)$. If F_0' is the form factor calculated using internal coordinates $r_i' = r_i - R_\mathrm{CM}$, $\Sigma_i r_i' \equiv 0$, then

$$\begin{aligned} F_0'(q^2) &= 1 - \tfrac{1}{6}\langle r'^2\rangle q^2 + \tfrac{1}{120}\langle r'^4\rangle q^4 + \ldots \\ &= F_0(q^2) f_\mathrm{CM}(q^2) \\ &= (1 - \tfrac{1}{6}\langle r^2\rangle q^2 + \tfrac{1}{120}\langle r^4\rangle q^4 + \ldots) \\ &\quad \times (1 + \alpha_2 q^2 + \alpha_4 q^4 + \ldots), \end{aligned} \quad (3)$$

where $\langle r^n\rangle \equiv A^{-1}\langle \Sigma_i r_i^n\rangle$. We immediately get

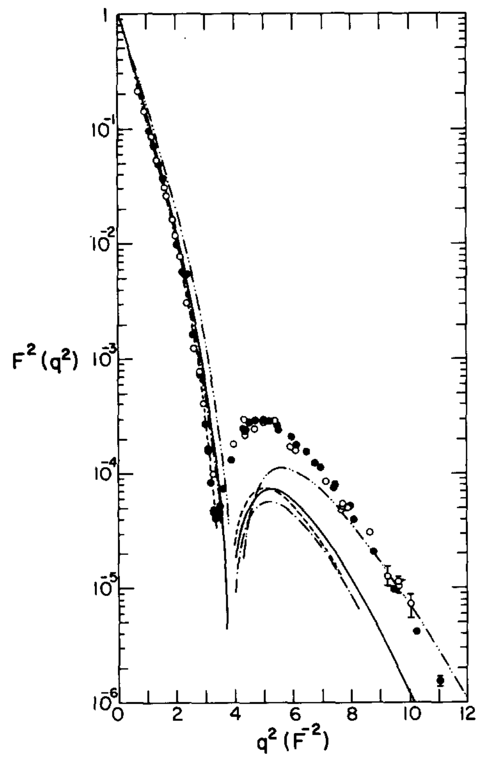

Fig. 1. Elastic form factor. Lines are projected HF result using a large basis with b = 1.539 fm (–) and b = 1.663 fm (– • –); and the small basis with the same values for b(– •• – and – – –). Data are from refs. [1], [2] and [3].

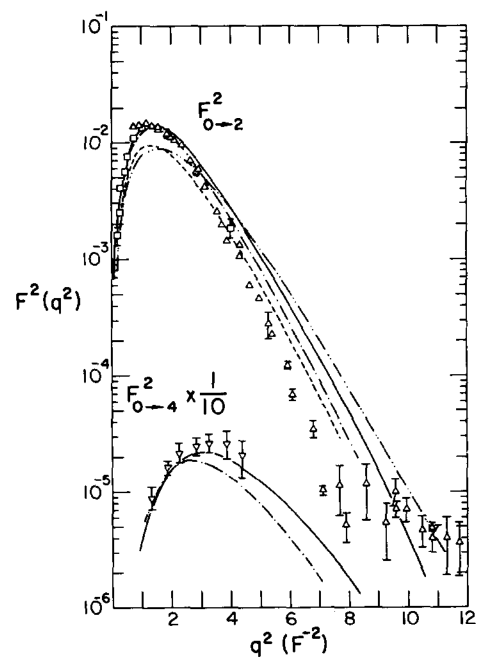

Fig. 2. Inelastic form factors for $0_1^+ \to 2_1^+$ and $0_1^+ \to 4_1^+$ transitions. For legend for lines see caption of fig. 1. Data are from refs. [1] and [2] (△), and [17] (□, △), respectively.

$\alpha_2 = \frac{1}{6}(\langle r^2\rangle - \langle r'^2\rangle)$, etc., in terms of the expectation values $\langle r^2\rangle$, $\langle r'^2\rangle$, etc. Notice that whereas r^n is a one-body operator, r'^n is an n- (or A-, whichever is smaller) body operator *. This simply reflects the fact that $e^{i\boldsymbol{q}\cdot\boldsymbol{r}'}$ is an A-body operator. However, unless high accuracy at very high momentum transfer is required, in general only α_2 is needed. In this case only the expectation value of a simple two-body operator, $\langle r'^2\rangle$, need be evaluated. This approximation is adopted in this work.

In fig. 1 the elastic Coulomb form factor for ^{12}C are shown. In fig. 2 are the inelastic $0_1^+ \to 2_1^+$ and $0_1^+ \to 4_1^+$ form factors. The only free parameter available is the size parameter of the oscillator wavefunction, b. The optium value, which leads to the lowest projected HF ground state energy, is b = 1.539 fm ($\hbar\omega$ = 17.5 MeV). For comparison the following calculations are also carried out: (i) with b = 1.663 fm ($\hbar\omega$ = 15.0 MeV) in the large basis, and (ii) with b = 1.539 fm and 1.663 fm in a "small" basis composed of only the first three (N = 0, 1, 2) major-shells. If the basis is sufficiently large, the result should be independent of b. In figs. 1 and 2 the large basis results (solid and dot–dash lines) show only a very weak dependence on b. This contrasts with the b-dependence of the small basis results (dot–dot–dash and dash lines). The differences in the b-dependence are also shown in table 1, where some projected HF results are listed. In the following we restrict our comments to the large basis results only.

The theoretical elastic form factor reproduces the data well up to the first diffraction minimum. After the minimum the agreement becomes significantly worse. It is known that simple [11] as well as more sophisticated *spherical* shell-model [12] wavefunctions are capable of producing better fits in this region. This is not surprising as Vinciguerra and Stovall [5] have shown that any deformation introduced into the ^{12}C wavefunction will suppress the elastic form factor in the region of the second maximum, thus wors-

* Here by a n-body operator we mean a linear combination of one- two-, . . . , and n-body.

Table 1
Some Hartree–Fock results.

	Large Basis		Small Basis	
	(1)	(2)	(1)	(2)
b(fm)	1.539	1.663	1.539	1.663
$\langle H \rangle$ (MeV)	−76.38	−76.05	−71.29	−71.98
E_{0^+} (MeV)	−81.46	−81.07	−75.58	−76.37
$E_{2^+} - E_{0^+}$ (MeV)	2.67	2.69	2.74	2.89
$E_{4^+} - E_{0^+}$ (MeV)	11.20	11.10	11.48	11.26
α_2(fm^2)**	0.0526	0.0544	0.0478	0.0564
$b^2/4A$ (fm^2)	0.0493	0.0576	0.0493	0.0576
r_{charge} (fm)†	2.429	2.460	2.329	2.477
$\langle Q_{20} \rangle$ (Cfm^2)††	− 8.01	− 8.21	− 5.33	− 6.23
$\langle Q_{40} \rangle$ (Efm^4)	13.9	15.7	0.61	0.90

** See text for definition.
† rms radius of protons w.r.t. CM and with proton radius folded in. Latest experimental value is [3] 2.46 ± 0.025 fm;
†† $Q_{\lambda\mu} = [4\pi/(2\lambda+1)]^{\frac{1}{2}} \sum_{i,\,\mathrm{proton}} r_i^\lambda Y_{\lambda\mu}(\Omega_i)$.

ening the fit. This situation which seemingly favours the spherical wavefunction over the deformed wavefunction for ^{12}C is counterbalanced by the situation in high-energy elastic p–^{12}C scattering. There the introduction of deformation improves the theoretical fit [13, 14]. The experimental e–^{12}C data, available up to $q^2 \sim 16\,\mathrm{fm}^{-2}$, do not show a second minimum. We predict a minimum at $q^2 \sim 17\,\mathrm{fm}^{-2}$.

The $0_1^+ \to 2_1^+$ data are also reproduced very well in the HF approximation up to $q^2 \sim 4\,\mathrm{fm}^{-2}$ but again the agreement with experiment becomes progressively less satisfactory for higher q^2 values. The only shell model calculation for this form factor is the particle–hole description of Gillet and Melkanoff [4]. In the random-phase approximation the quality of their fit is similar to ours. Data beyond $q^2 \sim 7.5\,\mathrm{fm}^{-2}$ suggest the existence of a diffraction minimum, which is not predicted here. Neither is it predicted in the shell model [4], the α-cluster model [6] or the hibridized cluster–shell model [6]. Vinciguerra and Stovall [5] showed that only a prolate deformation in ^{12}C can produce such a minimum. However, an oblate deformation is favored theoretically on energetic grounds and experimentally from analysis of α–^{12}C [15], and high-energy [13, 14] and low-energy [16] p–^{12}C data.

The data [17] for $0_1^+ \to 4_1^+$ are rather sparse, and these are well accounted for by the present calculation. The small basis results are not shown in fig. 2 as they are three orders of magnitude smaller than the data.

Recently Nakada et al. [17] reported an intrinsic state in the Nilsson-rotational model for the ground band of ^{12}C which can reproduce the e–^{12}C data as well as reported here. In their search for the intrinsic state the rms radius, and the intrinsic quadrupole and hexadecapole moments for ^{12}C were determined. Other matrix elements can then be calculated through the rotational model. In table 2 some such matrix elements are compared with those obtained in the projected HFA. The most interesting feature in table 2 is that the rotational model describes the HF results very well (up to $J = 4$, at which point the rotational band is truncated), except that to produce the same static moment or transition strength, a significantly *larger* (by 20 ~ 40%) intrinsic moment is needed in the rotational model. This feature is quite general in light nuclei with "rotational" characteristics [18].

We have shown that the projected HFA accounts for, in an essentially parameter-free way, the low energy data of the 0_1^+, 2_1^+ and 4_1^+ states in ^{12}C. It has failed

Table 2

	Hartree–Fock approximation		Rotational model †
b(fm)	1.539	1.663	1.50
$\langle Q_{20}\rangle$ (e fm^2)	– –8.01	– –8.21	– –10.1 [17]
$\langle Q_2\rangle_{2^+}$ (e fm^2)*	2.66	2.73	2.88
$\langle Q_2\rangle_{4^+}$ (e fm^2)	3.19	3.27	3.67
$\langle Q_2\rangle_{6^+}$ (e fm^2)	2.40	2.32	4.04
$\langle Q_2\rangle_{8^+}$ (efm^2)	1.41	1.17	4.25
B(E2; $0^+ \rightarrow 2^+$)(e^2 fm^4) **	35.4	37.3	40.6
B(E2; $2^+ \rightarrow 4^+$) (e^2 fm^4)	18.2	19.1	20.9
$\langle Q_{40}\rangle$ (e fm^4)	13.9	15.7	25.4 [17]
$\langle Q_4\rangle_{2^+}$ (e fm^4)	1.02	1.15	1.22
$\langle Q_4\rangle_{4^+}$ (e fm^4)	2.75	3.13	3.20
B(E4; $0^+ \rightarrow 4^+$) (e^2 fm^8)	393	503	462
B(E4; $2^+ \rightarrow 4^+$) (e^2 fm^8)	116	125	120

* $\langle Q_\lambda\rangle_J = \langle JJ|Q_{\lambda 0}|JJ\rangle$.

** $B(\mathrm{E}\lambda; J \rightarrow J') = [(2\lambda+1)/4\pi]\,|\langle J\|Q_\lambda\|J'\rangle|^2$; measured [2] value is $41.8 \pm 4\,\mathrm{e}^2\mathrm{fm}^4$.

† In the rotational model $\langle J'(K')\|Q_\lambda\|J(K)\rangle = \sqrt{(2J+1)/(2J'+1)}\,\langle J\lambda K' - K|J'K'\rangle\langle Q_{\lambda,K'-K}\rangle$, where K is the projection of J on the body-fixed Z-axis. In this work $K = K' = 0$.

and indeed is not expected, to account for the high-energy behaviour of these states. Among the any number of reasons behind this failure, the lack of two- or more-body correlations in the HF wavefunction is believed to be the most important. The effect of correlations on the elastic scattering has been studied [19]. However there are difficulties in its interpretation [20]. The inelastic $0_1^+ \rightarrow 2_1^+$ data seem even more challenging, as no theory has yet succeeded in reconciling the apparent oblate shape of ^{12}C and the existence of a diffraction minimum in the data.

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