CALCULATION OF THE VACUUM POLARIZATION IN NON-COVARIANT GAUGES INCORPORATING NIELSEN–KALLOSH GHOSTS

G. McKEON, S.B. PHILLIPS, S.S. SAMANT, T.N. SHERRY [1]

Department of Applied Mathematics, University of Western Ontario, London, Ontario, Canada N6A 5B9

H.C. LEE [a] and M.S. MILGRAM [b]

[a] *Theoretical Physics Branch, Atomic Energy of Canada Limited, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada K0J 1J0*

[b] *Applied Mathematics Branch, Atomic Energy of Canada Limited, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada K0J 1J0*

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The background field quantization procedure for pure YM theories is used in conjunction with non-covariant gauges characterized by the gauge fixing term $(1/2\alpha)n\cdot Q^a f_i^{ab} n\cdot Q^b$ where f_i^{ab} can assume the forms $f_\mathrm{I} = -\delta^{ab}$ (i.e. axial gauge), $f_\mathrm{II}^{ab} = (n\cdot \mathrm{D}(A))^{2ab}/(n^2)^2$ or $f_\mathrm{III} = \mathrm{D}^2(A)^{ab}/n^2$ (i.e. planar gauge) where $n^2 \neq 0$. Here A_μ^a and Q_μ^a represent respectively the classical background field and the quantum field. It is noted that if f_i^{ab} explicitly depends on the background field, then it is necessary to introduce Nielsen–Kallosh ghosts in addition to the expected Faddeev–Popov ghosts. Explicit calculations to one-loop order show that for f_II and f_III, the divergent part of the vacuum polarization is $[(\mathrm{i}/16\pi^2)C_2\delta^{ab}g^2/(2-\omega)]\frac{11}{3}(p^2\delta_{\mu\nu} - p_\mu p_\nu)$, while in the axial gauge the vacuum polarization is transverse but α- and n-dependent. The latter result – an apparent contradiction of Kallosh's theorem – is shown to arise due to the unconventional asymptotic behaviour of the vector propagator in the axial gauge.

1. Introduction. Capper and Leibbrandt [1] have examined the vacuum polarization tensor in a YM theory using Schwinger source term quantization with non-covariant gauge fixing terms of the general form

$$L_\mathrm{gf} = (1/2\alpha)\, n\cdot V^a f_i n\cdot V^a, \qquad i = \mathrm{I, II, III}, \tag{1}$$

where

$n^2 \neq 0,$

$$f_\mathrm{I} = -1, \qquad f_\mathrm{II} = (n\cdot\partial)^2/(n^2)^2, \qquad f_\mathrm{III} = \partial^2/n^2 .$$

For the planar gauge f_III, they find that in 2ω dimensions,

$$\begin{aligned}[\Pi_{\mu\nu}^{ab}(p)]_\mathrm{div} &= [(\mathrm{i}/16\pi^2)\, C_2\delta^{ab}g^2/(2-\omega)] \\ &\times [(\tfrac{11}{3} + 2\alpha)(p^2\delta_{\mu\nu} - p_\mu p_\nu) \\ &- (4\alpha/n^2)\, p^2 n_\mu n_\nu + 2\alpha p\cdot n p_\mu n_\nu + 2\alpha p\cdot n p_\nu n_\mu].\end{aligned} \tag{2}$$

Even though the vacuum polarization tensor is non-transverse and α- and n-dependent, they find that this result is consistent with the unusual form of the Taylor–Slavnov identities in this gauge. For the axial gauge f_I, they find

$$\begin{aligned}[\Pi_{\mu\nu}^{ab}(p)]_\mathrm{div} &= [(\mathrm{i}/16\pi^2)\, C_2\delta^{ab}g^2/(2-\omega)] \\ &\times \{\tfrac{11}{3}(p^2\delta_{\mu\nu} - p_\mu p_\nu) \\ &+ [4\alpha/3(n^2)^2]\,(p\cdot n p_\mu - p^2 n_\mu)(p\cdot n p_\nu - p^2 n_\nu)\}.\end{aligned} \tag{3}$$

[1] Permanent address: Department of Mathematical Physics, National University of Ireland, University College, Galway, Ireland.

The vacuum polarization tensor here is transverse but α- and n-dependent. For $f_{\mathrm{II}} = (n\cdot\partial)^2/(n^2)^2$, they find

$$[\Pi^{ab}_{\mu\nu}(p)]_{\mathrm{div}} = [(\mathrm{i}/16\pi^2)\, C_2 \delta^{ab} g^2/(2-\omega)] \times \tfrac{11}{3}(p^2\delta_{\mu\nu} - p_\mu p_\nu). \qquad (4)$$

Henceforth we will set $n^2 = 1$ for simplicity.

In this paper, we use the background field quantization procedure [2,3] to study YM theories in non-covariant gauges similar to those found in eq. (1). If the gauge field V^a_μ is broken up into the classical (A^a_μ) and the quantum (Q^a_μ) parts, the gauge fixing term in the lagrangian is

$$L_{\mathrm{gf}} = (1/2\alpha)\, n\cdot Q^a f_i^{ab}\, n\cdot Q^b, \qquad i = \mathrm{I, II, III}, \qquad (5)$$

where

$$n^2 \neq 0, \qquad f_{\mathrm{I}}^{ab} = -\delta^{ab},$$

$$f^{ab}_{\mathrm{II}} = [n\cdot \mathrm{D}(A)]^{2\,ab}, \qquad f^{ab}_{\mathrm{III}} = \mathrm{D}^2(A)^{ab}.$$

The covariant derivative in the background field is defined by

$$\mathrm{D}^{ab}_\mu(A) = \partial_\mu \delta^{ab} + g f^{apb} A^p_\mu. \qquad (6)$$

It is important to note that the gauge fixing term given in eq. (5) does not break type I gauge invariance defined by

$$\delta A^a_\mu = \mathrm{D}^{ab}_\mu(A)\,\Lambda^b, \qquad \delta Q^a_\mu = g f^{acb} Q^c_\mu \Lambda^b, \qquad (7)$$

but does break type II gauge invariance defined by

$$\delta A^a_\mu = 0, \qquad \delta Q^a_\mu = \mathrm{D}^{ab}_\mu(A+Q)\,\Lambda^b. \qquad (8)$$

The Feynman rules for the theory are developed in section 2 by use of the path integral quantization procedure. In addition to the usual Faddeev–Popov ghost fields c and $\bar{c}$, we find that for f_{II} and f_{III} it is necessary to introduce Nielsen–Kallosh ghosts – complex anticommuting scalar ghost fields ω and $\bar{\omega}$, and a real commuting scalar ghost field γ. Nielsen and Kallosh [4] have shown that similar ghost fields arise in supergravity.

The divergent parts of the one-loop corrections to the vacuum polarization tensor for f_{I}, f_{II} and f_{III} are computed in section 3. For f_{I} it is found that the vacuum polarization is identical to Capper and Leibbrandt's result in eq. (3). This appears to contradict a theorem of Kallosh [5], which states that if the effective lagrangian for a pure YM theory is type I gauge invariant, then the counterterms are independent of the gauge fixing condition, even off mass shell, in background field quantization. On the other hand for f_{II} and f_{III} our results are consistent with the predictions of Kallosh's theorem. In section 4 we discuss why Kallosh's theorem is upheld for the cases f_{II} and f_{III}, while being apparently contradicted by f_{I} for $\alpha \neq 0$.

2. *Quantization procedure.* First of all we introduce the Feynman amplitude

$$F[A^a_\mu] = \int \mathrm{d}Q \exp\left(\mathrm{i}\int \mathrm{d}^4x\,(-\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu})\right), \qquad (9)$$

where

$$F^a_{\mu\nu} = \partial_\mu (A+Q)^a_\nu - \partial_\nu (A+Q)^a_\mu + g f^{abc} (A+Q)^b_\mu (A+Q)^c_\nu.$$

If we follow an argument given by 't Hooft [6], eq. (9) becomes

$$F[A^a_\mu] = \int \mathrm{d}Q\, \mathrm{d}c\, \mathrm{d}\bar{c}\, \delta(n\cdot Q^a - b^a) \times \exp\left(\mathrm{i}\int \mathrm{d}^4x\,[-\tfrac{1}{4} F^a_{\mu\nu}F^{a\mu\nu} + \bar{c}^a n\cdot \mathrm{D}^{ab}(A+Q)\, c^b]\right), \qquad (10)$$

where we use the gauge fixing condition

$$n\cdot Q^a = 0, \qquad (11)$$

and c and $\bar{c}$ are the standard Faddeev–Popov ghosts fields. Eq. (10) is independent of the field b^a, so we can integrate over all fields b^a with an appropriate weight factor in order to exponentiate the gauge fixing term in eq. (10). We multiply eq. (10) by

$$1 = k\int \mathrm{d}b^a \exp\left(\mathrm{i}\int \mathrm{d}^4x\,[(1/2\alpha)\, b^a f_i^{ab} b^b] \times (\mathrm{Det}\,\alpha^{-1} f_i^{ab})^{1/2}\right), \qquad (12)$$

leading to the result

$$F[A^a_\mu] = \int \mathrm{d}Q\, \mathrm{d}c\, \mathrm{d}\bar{c}\, (\mathrm{Det}\, f_i^{ab})^{1/2} \times \exp\left(\mathrm{i}\int \mathrm{d}^4x\,[-\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + (1/2\alpha)\, n\cdot Q^a f_i^{ab} n\cdot Q^b + \bar{c}^a n\cdot \mathrm{D}^{ab}(A+Q)\, c^b]\right). \qquad (13)$$

The factor of $(\mathrm{Det}\, f_i^{ab})^{1/2}$, where f_i^{ab} is given in eq. (5), is an innocuous scale factor *except in those cases where f_i^{ab} explicitly depends on A_μ^a*. In such cases, it can be exponentiated into the action in the following way:

$$(\mathrm{Det}\, f_i^{ab})^{1/2} = (\mathrm{Det}\, f_i^{ab})(\mathrm{Det}\, f_i^{ab})^{-1/2}$$
$$= \int \mathrm{d}\omega\, \mathrm{d}\bar{\omega} \exp\left(\mathrm{i}\int \mathrm{d}^4x\, \bar{\omega}^a f_i^{ab}\, \omega^b\right)$$
$$\times \int \mathrm{d}\gamma \exp\left(\mathrm{i}\int \mathrm{d}^4x\, \tfrac{1}{2}\, \gamma^a f_i^{ab} \gamma^b\right), \qquad (14)$$

where ω and $\bar{\omega}$ are complex anticommuting scalars, and γ is a real commuting scalar. The ghost fields ω, $\bar{\omega}$ and γ are the YM counterparts of the Nielsen–Kallosh ghosts in supergravity ‡1.

Using these results, the generating functional for the Green function is defined by

$$Z[A_\mu^a, \eta_\mu^a] = \int \mathrm{d}Q\, \mathrm{d}c\, \mathrm{d}\bar{c}\, \mathrm{d}\omega\, \mathrm{d}\bar{\omega}\, \mathrm{d}\gamma$$
$$\times \exp\Big(\mathrm{i}\int \mathrm{d}^4x\, [-\tfrac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$
$$+ (1/2\alpha)\, n\cdot Q^a f_i^{ab}\, n\cdot Q^b + \bar{c}^a n\cdot \mathrm{D}^{ab}(A+Q)\, c^b$$
$$+ \bar{\omega}^a f_i^{ab}\, \omega^b + \tfrac{1}{2}\gamma^a f_i^{ab} \gamma^b + \eta_\mu^a Q^{a\mu}]\Big), \qquad (15)$$

where η_μ^a is the source term for the quantum field Q_μ^a.

In computing the one-loop Green function with two external classical gauge fields, A_μ, we use the Feynman rules derived from the effective lagrangian of eq. (15) for the appropriate f_i^{ab}.

3. The vacuum polarization tensor to one-loop order. We now consider the one-particle irreducible (1PI) Green function with two external classical gauge fields. This Green function has been evaluated to two-loop order in covariant gauges where tha gauge fixing term is

$$L_{\mathrm{gf}} = -(1/2\alpha)\, [\mathrm{D}_\mu^{ab}(A)\, Q_\mu^b]^2 \qquad (16)$$

for the specific case $\alpha = 1$ by Abbott [3] and for general α by Capper and Maclean [7]. They both find

$$[\Pi_{\mu\nu}^{ab}(p)]_{\mathrm{div}} = [(\mathrm{i}/16\pi^2)\, C_2 \delta^{ab} g^2/(2-\omega)]$$
$$\times [\tfrac{11}{3} + (g^2 C_2/16\pi^2)\, \tfrac{17}{3}]\,(p^2\delta_{\mu\nu} - p_\mu p_\nu). \qquad (17)$$

This result is independent of the gauge parameter α, and hence agrees with Kallosh's theorem [5].

We now examine the same Green function for the gauge fixing term of eq. (5), computing $[\Pi_{\mu\nu}^{ab}]_{\mathrm{div}}$ for each of the three cases, f_{I}, f_{II} and f_{III}. For $i = \mathrm{I}$ [i.e., $L_{\mathrm{gf}} = (1/2\alpha)\,(n\cdot Q^a)^2$] the Feynman rules pertinent to our calculation of the vacuum polarization tensor are identical to those used by Capper and Leibbrandt [1] in their calculation since in eq. (13) $\mathrm{Det}\, f_{\mathrm{I}}^{ab} = 1$. Consequently the result in this case is identical to their result given in eq. (3). This result is apparently only consistent with Kallosh's theorem in the limit $\alpha \to 0$. It was just this limit that was used in ref. [8] to calculate $[\Pi_{\mu\nu}^{ab}]_{\mathrm{div}}$ for the explicit verification of Kallosh's theorem in the axial gauge. However Kallosh's theorem does not hold in the axial gauge for $\alpha \neq 0$.

Now we turn to the case of f_{II} [i.e., $L_{\mathrm{gf}} = (1/2\alpha)\, n\cdot Q^a\, (n\cdot \mathrm{D}(A))^{2\,ab}\, n\cdot Q^b$]. To calculate $[\Pi_{\mu\nu}^{ab}(p)]_{\mathrm{div}}$ here, we recall that Capper and Leibbrandt have evaluated the vacuum polarization tensor for $L_{\mathrm{gf}} = (1/2\alpha)\, n\cdot Q^a (n\cdot\partial)^2\, n\cdot Q^a$ in conventional quantization. Hence we need only evaluate the Nielsen–Kallosh bubble diagrams given in fig. 1, and compute the contributions of the new AQQ vertices generated by the gauge fixing term in the effective lagrangian. The divergent parts of the Nielsen–Kallosh bubble diagrams involve only integrals of the type

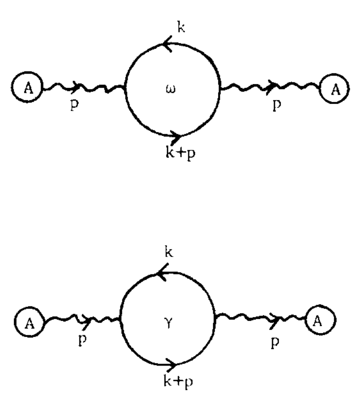


Fig. 1.

‡1 The exponentiation of the $(\mathrm{Det}\, f)^{1/2}$ factor for background field quantized YM theory was first discussed by de Witt [2]. However his discussion centered on f^{-1} rather than f, so that he used real commuting scalar ghost fields χ, $(\mathrm{Det}\, f^{-1})^{-1/2} = \int \mathrm{d}\chi \exp[\mathrm{i}\int \mathrm{d}^4x\, (\tfrac{1}{2}\chi^a f^{-1\,ab}\chi^b)]$.

$$\int d^{2\omega}k \frac{1}{(n\cdot k)^2\,[n\cdot(k+p)]^2} \begin{pmatrix} (n\cdot k)^2 \\ n\cdot k\, n\cdot p \\ n\cdot(k+p)\, n\cdot k \\ (n\cdot p)^2 \end{pmatrix},$$

which are zero in dimensional regularization [9].

The new AQQ vertices arise from that part of L_{gf} given by

$$(1/2\alpha) n\cdot Q^a (n\cdot\partial\, g f^{apb} n\cdot A^p + n\cdot A^p\, g f^{apb} n\cdot\partial) n\cdot Q^b .$$

In the bubble diagram of fig. 2, these vertices give rise to integrals of the type

$$\int d^{2\omega}k \frac{1}{(n\cdot k)^2\,[n\cdot(k+p)]^2} \begin{pmatrix} n\cdot k \\ n\cdot p \end{pmatrix},$$

which again have vanishing divergent parts. The appropriate tadpole diagrams are also zero [10]. Since the extra terms in $[\Pi^{ab}_{\mu\nu}(p)]_{\mathrm{div}}$ in the background approach have been shown to have vanishing divergences, $[\Pi^{ab}_{\mu\nu}(p)]_{\mathrm{div}}$ is given by the Capper–Leibbrandt result of eq. (4),

$$[\Pi^{ab}_{\mu\nu}]_{\mathrm{div}} = [(\mathrm{i}/16\pi^2)\, C_2\delta^{ab} g^2/(2-\omega)] \times \tfrac{11}{3}(p^2\delta_{\mu\nu} - p_\mu p_\nu).$$

Clearly this result is consistent with Kallosh's theorem.

Finally we consider the case of f_{III} where

$$L_{\mathrm{gf}} = (1/2\alpha)\, n\cdot Q^a D^2(A)^{ab} n\cdot Q^b .$$

In this case the extra contributions to $[\Pi^{ab}_{\mu\nu}(p)]_{\mathrm{div}}$ do not all vanish trivially and so detailed calculations are required. The relevant Feynman rules are given in fig. 3, and the diagrams to be computed are given in fig. 4.

The tadpole diagrams, in fig. 4d–f can be shown to be zero [9,10]. The bubble diagram with the Faddeev-Popov ghost loop of fig. 4b can be shown to vanish

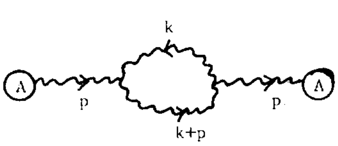


Fig. 2.

using the techniques of Capper and Leibbrandt [9]. The Nielsen–Kallosh ghost contributions of fig. 4c give

$$I_1 = [(\mathrm{i}/16\pi^2)\, C_2\delta^{ab} g^2/(2-\omega)]\, \tfrac{1}{6}\, (p^2\delta_{\mu\nu} - p_\mu p_\nu). \tag{18}$$

The contribution of fig. 2a is given by

$$-\tfrac{1}{2} g^2 C_2\delta^{ab} \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{k^2(k+p)^2}$$
$$\times \left[\left(\delta_{\alpha\beta} - \frac{(n_\alpha k_\beta + n_\beta k_\alpha)}{n\cdot k} + \frac{k_\alpha k_\beta n^2(1+\alpha)}{(k\cdot n)^2}\right)\right.$$
$$\times \left(\delta_{\lambda\sigma} - \frac{n_\lambda (k+p)_\sigma + n_\sigma (k+p)_\lambda}{n\cdot(k+p)} + \frac{(k+p)_\lambda (k+p)_\sigma n^2 (1+\alpha)}{[n\cdot(k+p)]^2}\right)$$
$$\times [\delta_{\mu\lambda}(-2p-k)_\alpha + (\delta_{\alpha\lambda} + \alpha^{-1} n_\alpha n_\lambda/n^2)(2k+p)_\mu + \delta_{\mu\alpha}(-k+p)_\alpha]$$
$$\times [\delta_{\nu\sigma}(2p+k)_\beta + (\delta_{\beta\sigma} + \alpha^{-1} n_\beta n_\sigma/n^2)(-2k-p)_\nu \left. + \delta_{\nu\beta}(k-p)_\sigma]\right]. \tag{19}$$

Explicit calculation of the integrals in (19) shows that the α- and n-dependences cancel out, giving the result

$$I_2 = [(\mathrm{i}/16\pi^2)\, C_2\delta^{ab} g^2/(2-\omega)]\, \tfrac{7}{2}\, (p^2\delta_{\mu\nu} - p_\mu p_\nu). \tag{20}$$

Adding (18) and (20), we find

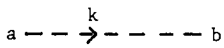
$$[\Pi^{ab}_{\mu\nu}(p)]_{\mathrm{div}} = [(\mathrm{i}/16\pi^2)\, C_2\delta^{ab} g^2/(2-\omega)] \times \tfrac{11}{3}(p^2\delta_{\mu\nu} - p_\mu p_\nu). \tag{21}$$

This is obviously independent of α and n, and hence consistent with Kallosh's theorem. It is the extra contribution to the three-point vertex arising from the gauge fixing term in the effective lagrangian and the Nielsen–Kallosh ghost contributions that are responsible for restoring the result of ref. [1] for the vacuum polarization in the conventional quantization procedure to the simple form given in eq. (21).

4. Kallosh's theorem and counterterms. Kallosh's theorem [5] states that if the gauge fixing term main-

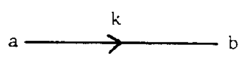
$$\frac{-i\,\delta^{ab}}{i(k^2+i\varepsilon)}\left[\delta_{\mu\nu} - \frac{(p_\mu n_\nu + p_\nu n_\mu)}{p\cdot n} + \frac{p_\mu p_\nu n^2(1+\alpha)}{(p\cdot n)^2}\right]$$

Gauge vector propagator



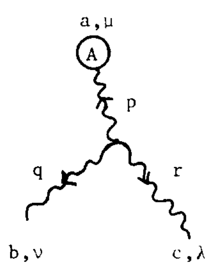
$$\frac{i\,\delta^{ab}}{n\cdot k}$$

Fadeev-Popov ghost propagator



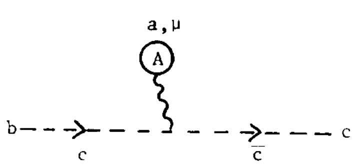
$$\frac{i\,\delta^{ab}}{k^2}$$

Nielsen-Kallosh ghost propagator



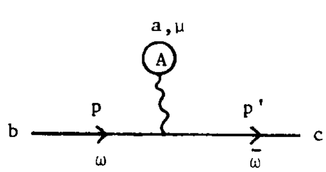
$$gf^{abc}\left[\delta_{\mu\lambda}(p-r)_\nu + \delta_{\nu\lambda}(r-q)_\mu + \delta_{\mu\nu}(q-p)_\lambda + \frac{1}{\alpha}\frac{n_\nu n_\lambda}{n^2}(r-q)_\mu\right]$$

Three point vertex



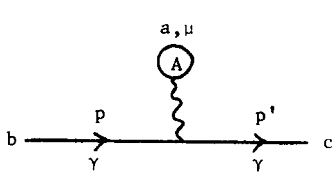
$$-igf^{abc}\,n_\mu$$

Gauge vector-FP ghost vertex



$$gf^{abc}(p+p')_\mu$$

Gauge vector-NK(anticommuting) ghost vertex



$$gf^{abc}(p+p')_\mu$$

Gauge vector-NK(commuting) ghost vertex

Fig. 3.

tains explicit type I gauge invariance, then for a pure YM gauge theory the counterterms necessary for renormalization are independent of the gauge choice. However, the proof of the theorem is only valid if, for the gauge fixing term, all diagrams with five or more external legs are divergence free.

To apply this to our evaluation of the vacuum polarization tensor, we first note that for f_i, $i = \mathrm{I, II}$ and III, explicit type I gauge invariance is maintained. In fact in all three cases, the naive Ward identity associated with type I gauge invariance [11],

$$\mathrm{D}^{ab}_{\mu}(A)\,\delta\Gamma/\delta A^{b}_{\mu}(x) = 0, \tag{22}$$

is satisfied since in background field quantization, type I gauge invariance is maintained. This is further discussed in refs. [11,12].

To determine the validity of Kallosh's theorem, we look at the form of the vector propagator in all three cases [1]:

$$f_{\mathrm{I}}:\quad G^{ab}_{\mu\nu}(p) = \frac{\delta^{ab}}{\mathrm{i}(p^2+\mathrm{i}\epsilon)}\left(\delta_{\mu\nu} - \frac{(p_\mu n_\nu + p_\nu n_\mu)}{p\cdot n} + \frac{p_\mu p_\nu n^2}{(p\cdot n)^2} + \alpha\,\frac{p_\mu p_\nu p^2}{(p\cdot n)^2}\right), \tag{22a}$$

$$f_{\mathrm{II}}:\quad G^{ab}_{\mu\nu}(p) = \frac{\delta^{ab}}{\mathrm{i}(p^2+\mathrm{i}\epsilon)}\left(\delta_{\mu\nu} - \frac{(p_\mu n_\nu + p_\nu n_\mu)}{p\cdot n} + \frac{p_\mu p_\nu n^2}{(p\cdot n)^2} + \alpha\,\frac{p_\mu p_\nu p^2 (n^2)^2}{(p\cdot n)^4}\right), \tag{22b}$$

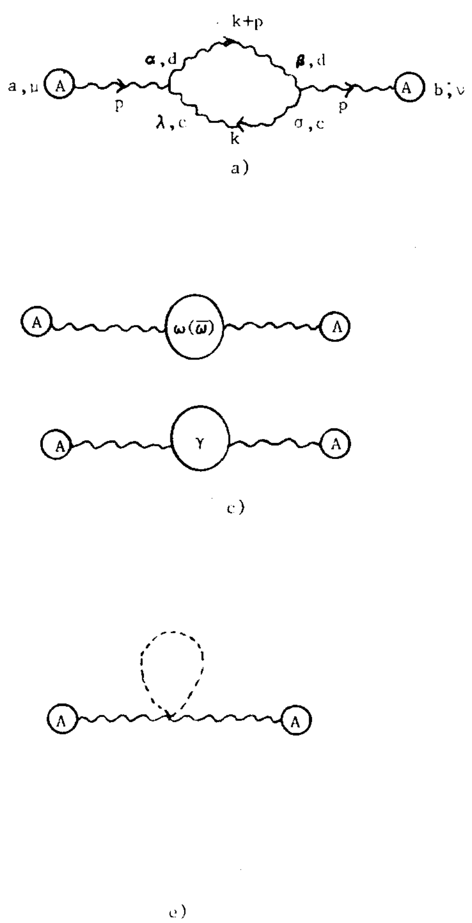


Fig. 4.

$$f_{\mathrm{III}}:\quad G^{ab}_{\mu\nu}(p)=\frac{\delta^{ab}}{\mathrm{i}(p^2+\mathrm{i}\epsilon)}\left(\delta_{\mu\nu}-\frac{(p_\mu n_\nu+p_\nu n_\mu)}{p\cdot n}+\frac{p_\mu p_\nu n^2(1+\alpha)}{(p\cdot n)^2}\right). \tag{22c}$$

These propagators have the conventional $O(1/p^2)$ behaviour for large p^2 except in the axial gauge case for $\alpha \neq 0$. In this case, the propagator is of order $\sim O(1)$ for large p^2. Thus diagrams with more than four external legs are not expected to be convergent in general. Hence Kallosh's theorem is not applicable to the background field quantization in the axial gauge for $\alpha \neq 0$, and the counterterms are not expected to be independent of the gauge condition. For the other two cases, diagrams with more than four external legs are convergent. Thus Kallosh's theorem is expected to hold in these cases. This is exactly what was found in section 3.

The gauge dependence of the counterterms and the $O(1)$ behaviour of the gauge propagator for large p^2 in the axial gauge poses problems for the renormalizability of a YM theory in background field quantization in that gauge because (i) the counterterms involve the introduction of new interactions not present in the original lagrangian; (ii) an infinite number of counterterms are required to make the theory finite to all orders. These points are further discussed in ref. [12].

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