GLUON EFFECTS IN CHARMED MESON DECAYS

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The emission of gluons from the initial and final states generates color-singlet and -octet terms in the effective lagrangian that can provide a quantitative description of inclusive and two-body hadronic decays of the charmed D^0, D^+ and F^+ mesons.

Recent experiments [1–3] have revealed many interesting and puzzling features of the decays of charmed mesons. In particular these experiments indicate the lifetimes of the mesons are very different: $\tau_{D^+} \approx 5\tau_{F^+} \approx 10\tau_{D^0}$. In early quark models [4] it was expected that the D^+ and D^0 mesons would have the same lifetimes because their constituent light antiquark was assumed to play merely the role of a spectator while the c quark decayed into light quarks instead of annihilating with the antiquark in the decay process. Another remarkable experimental finding is that the partial width of the $D^0 \to \bar{K}^0\pi^0$ decay is of the same order as that of the $D^0 \to K^-\pi^+$ decay, instead of being more than one order of magnitude smaller as predicted in the spectator model. Thus it seems that if the quark model is to be saved, the annihilation or non-spectator process, which was previously thought to be weak due to the helicity suppression mechanism [4], must play an important role in some decays.

A large annihilation contribution would follow from the assumption that charmed mesons contain not only the $c\bar{q}$ component but also gluons; then $c\bar{q}$ could carry spin 1 and annihilation would proceed unimpeded [5]. Alternatively the initial $c\bar{q}$ pair could acquire spin 1 by emitting a gluon thus again allowing annihilation to occur [6,7]. However as long as we do not know how to estimate the gluonic component in the meson wavefunction, because perturbation theory is not reliable here, the above schemes are unlikely to give quantitative or even semi-quantitative predictions. Yet another approach is to parametrize the annihilation contributions by a single parameter without regard to gluons. It is then possible to explain the Cabibbo-allowed decays of D^+ and D^0 but the disagreement with the data persists at the Cabibbo-suppressed level, and the model gives unrealistically large widths for F^+ decays unless flavor-SU(3) is broken.

It has also been suggested that strong final state interactions acting on the hadrons in the form of resonances can create the observed effects [8]. Although it is clear that final state interactions must contribute, it is, however, unlikely that they are strong enough to produce the large difference in the D^+, D^0 lifetimes; furthermore it is difficult to be specific in this approach because of the large numbers of unknown amplitudes or resonances needed to reproduce all favored and suppressed decays of D^+, D^0 and F^+.

In this note we will remain within the interacting quark model and stay close to the QCD ansatz. But in-

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stead of letting the hadron wavefunctions be contaminated by gluon admixtures we consider them to be in pure color singlet states and allow gluons to be emitted during the decay process. Our approach is similar in spirit to that of ref. [7], but we carry our analysis further. We show that by allowing gluons to be emitted in the initial and final states, color-singlet and -octet terms in the effective lagrangian are generated that allow all inclusive and two-body hadronic decay data to be quantitatively accounted for.

We start with the weak, charm-changing ($\Delta C = -1$) interaction:

$$H_W = 2\sqrt{2}\,G[f_+(\bar{s}'_L\gamma_\mu c_L)(\bar{u}_L\gamma^\mu d'_L) + f_-(\bar{u}_L\gamma_\mu c_L)(\bar{s}'_L\gamma^\mu d'_L)], \qquad (1)$$

with the following notations: $q_L = \frac{1}{2}(1+\gamma_5)q$, $d' = d \times \cos\theta_c + s\sin\theta_c$, $s' = -d\sin\theta_c + s\cos\theta_c$, and $f_\pm = \frac{1}{2} \times (c_+ \pm c_-)$. Here θ_c is the Cabibbo angle (we neglect complications [9] arising from the proliferation of Cabibbo angles in the six- or more-quark models), and c_+, c_- are the renormalized coefficients belonging to the **84** and **20** SU(4) representations, respectively. In calculating quark matrix elements one uses Fierz reordering of spinors; alternatively one can first effect a Fierz transformation on the interaction operator:

$$H_W \to 2\sqrt{2}\,G[\chi_+(\bar{s}'_L\gamma_\mu c_L)(\bar{u}_L\gamma^\mu d'_L) + 2f_-(\bar{s}'_L\tfrac{1}{2}\lambda^a\gamma_\mu c_L)(\bar{u}_L\tfrac{1}{2}\lambda^a\gamma^\mu d'_L)] \qquad (2a)$$

$$\to 2\sqrt{2}\,G[\chi_-(\bar{u}_L\gamma_\mu c_L)(\bar{s}'_L\gamma^\mu d'_L) + 2f_+(\bar{u}_L\tfrac{1}{2}\lambda^a\gamma_\mu c_L)(\bar{s}'_L\tfrac{1}{2}\lambda^a\gamma^\mu d'_L)], \qquad (2b)$$

where $\chi_\pm = \frac{1}{3}(2c_+ \pm c_-)$ and λ^a ($a = 1,\ldots,8$) are the generators of the color-SU(3) group.

We first consider the spectator mechanism. Fig. 1a represents a color-connected charm decay and gives rise to a factor χ_+; fig. 1b represents a color-disconnected c decay and gives rise to a factor χ_-. In calculating these diagrams, as well as all other diagrams, we take as a working assumption that all quark–antiquark pairs have the same distribution function $\psi(\boldsymbol{r})$ of the quark–antiquark distance $\boldsymbol{r}$. Assuming further that bound quarks have small momenta it can be shown [10] that, in the lowest order, the spectator amplitudes depend on two factors: the first is the meson wavefunction at zero $q\bar{q}$ distance, $\psi(0)$, which we associate with the leptonic meson decay constant having the dimension of mass, F_M, and the second is the transition density $\int d^3r \exp(i\boldsymbol{p}\cdot\boldsymbol{r})\,\psi^*_M(\boldsymbol{r})\,\psi_D(\boldsymbol{r})$ calculated at the final relative momentum $\boldsymbol{p}$ and which we associate with the dimensionless form factor f^c of the semi-leptonic D-decay. For example, the amplitude for $D^0 \to \bar{K}^0\pi^0$ can be expressed in the following factorized form:

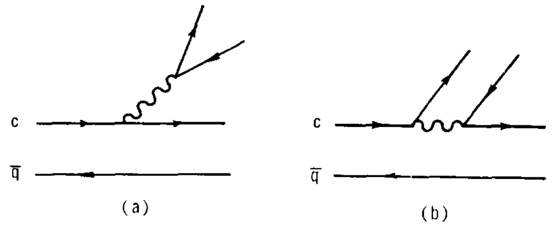

Fig. 1. Spectator diagrams. The final $q-\bar{q}$ pairs are (a) color-connected and (b) color-disconnected. The wavy line represents a QCD renormalized W-boson that carries the coefficients c_+ and c_-.

$$(4/2\sqrt{2}\,m_D^2 G\cos^2\theta_c)\langle\bar{K}^0\pi^0|H_W|D_0\rangle_{sp}$$

$$= (4\chi_-/m_D^2)\langle\bar{K}^0|\bar{s}_L\gamma_\mu d_L|0\rangle\langle\pi^0|\bar{u}_L\gamma_\mu c_L|D_0\rangle$$

$$= (\chi_-/\sqrt{2}m_D^2)F_K[(m_D^2 - m_\pi^2)f_+^c(m_K^2) + m_K^2 f_-^c(m_K^2)]$$

$$\approx (1/\sqrt{2})\,\chi_- F_K f^c, \qquad (3a)$$

where $f^c = f_+^c(0) \approx f_+^c(m_M^2)$, and the charmless meson mass m_M is neglected compared with m_D. More generally the spectator amplitudes can be written in the form

$$A_{sp} = a\chi_\pm F_M f^c, \qquad (3b)$$

where a is determined by the structure of the pseudoscalar meson as an SU(3) $(q\bar{q})$ state.

We turn next to the non-spectator mechanism. When gluons are allowed to enter the picture, not only the color-singlet but also the color-octet terms of the hamiltonian (2) contribute. Consequently, similarly to (3b), the amplitudes for the annihilation processes are of the form

$$A_{ns} = \chi_+ S + f_- O, \quad \text{or} \quad A_{ns} = \chi_- S + f_+ O. \qquad (4a,b)$$

Because the singlet and octet contributions, S and O, depend on the propagators of the intermediate quarks, it is useful to show explicitly the numbers of quarks, n_s and n_0, in the participating mesons that can contribute to the non-spectator processes. Whenever a strange quark is involved, the corresponding weight is reduced by a mass factor $\mu = m_u/m_s \approx 0.62$. Processes where gluons are emitted from a c quark, being of order m_u/m_c, are ignored. In this way flavor-SU(3) is naturally broken. Thus we may rewrite (4a,b) as

$$A_{ns} = \chi_\pm n_s g_s + f_\mp n_0 g_0, \tag{5}$$

where (n_s, n_0) is equal to $(1+\mu, 1)$, (0,0) and $(2,\mu)$ for the Cabibbo-favored decays of D^0, D^+ and F^+, respectively, it is equal to (2,1) or $(2\mu, 1)$, (2,1) and $(1+\mu, \mu)$ for the corresponding Cabibbo-suppressed decays, and χ_+ (χ_-) always appears with f_- (f_+). In the lowest order, with a single gluon exchange, the octet decay arises from the process depicted in fig. 2a, and the singlet decay is represented by fig. 2b. In our pure-

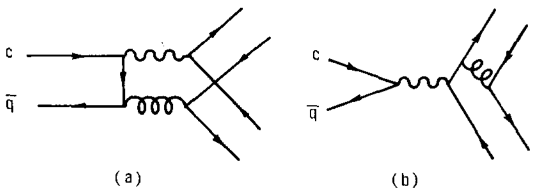

Fig. 2. Annihilation or non-spectator diagrams. The color-octet term (a) arises from gluons (coiled line) emitted from the initial state. In the color-singlet term (b) gluons are exchanged only in the final state to create the extra $q\bar{q}$ pair; gluons emitted from the final antiquark are considered but not shown here.

Table 1
Two-body nonleptonic decays of charmed mesons.

Decay branch	A [a]	Calc. widths (10^{11} s^{-1})	Expt. widths [c] (10^{11} s^{-1})
$D^0 \to K^-\pi^+$	$1 + (1+\mu) x g_s + y_1 g_0$	3.0 [b]	$(3.0 \pm {}^{2.2}_{2.6})$
$\to \bar{K}^0\pi^0$	$2^{-1/2}[xf - (1+\mu) x g_s - y_1 g_0]$	2.4 [b]	$(2.2 \pm {}^{2.6}_{2.1})$
$\to \bar{K}^0\eta$	$6^{-1/2}[xf - (1+\mu) x g_s - y_1 g_0]$	0.8	
$D^+ \to \bar{K}^0\pi^+$	$1 + xf$	0.22 [b]	$(0.22 \pm {}^{0.25}_{0.15})$
$F^+ \to \bar{K}^0 K^+$	$xf + 2g_s + \mu y_2 g_0$	1.5	
$\to \eta\pi^+$	$(2/3)^{1/2}[-1 + 2g_s + \mu y_2 g_0]$	1.9	
$D^0 \to K^-K^+$	$t[f + 2\mu x g_s + y_1 g_0]$	0.15	$(0.40 \pm {}^{0.46}_{0.36})$
$\to \bar{K}^0 K^0$	$-2t(1-\mu) x g_s$	0.01	
$\to \pi^-\pi^+$	$t(1 + 2x g_s + y_1 g_0)$	0.12	$(0.10 \pm {}^{0.15}_{0.09})$
$\to \pi^0\pi^0$	$2^{-1/2} t[x - (2x g_s + y_1 g_0)]$	0.12	
$\to \eta\pi^0$	$3^{-1/2} t[\frac{1}{2}x(1-3f) + 2x g_s + y_1 g_0]$	0.06	
$\to \eta\eta$	$2^{-1/2} t[-xf + \frac{2}{3}(4\mu-1) x g_s + y_1 g_0]$	0.17	
$D^+ \to \bar{K}^0 K^+$	$t[f - (2g_s + y_2 g_0)]$	0.05	$(0.05 \pm {}^{0.08}_{0.03})$
$\to \pi^0\pi^+$	$2^{-1/2} t(1 + x)$	0.01	
$\to \eta\pi^+$	$6^{-1/2} t[(1+3xf) + 4g_s + 2y_2 g_0]$	<0.01	
$F^+ \to K^0\pi^+$	$t[-1 + (1+\mu) g_s + \mu y_2 g_0]$	0.09	
$\to K^+\pi^0$	$2^{-1/2} t[x + (1+\mu) g_s + \mu y_2 g_0]$	0.01	
$\to K^+\eta$	$6^{-1/2} t[(2+3x) f + (1+\mu) g_s + \mu y_2 g_0]$	<0.01	

a) We use $c_- = 2.1$, $c_+ = 0.35$, with $f_\pm = (c_+ \pm c_-)/2$, $\chi_\pm = (2c_+ \pm c_-)/3$. Then $x = \chi_-/\chi_+ = -0.50$, $y_1 = f_+/\chi_+ = 1.31$, $y_2 = f_-/\chi_+ = -0.94$, $f = F_K/F_\pi = 1.23$, $t \approx \tan\theta_c = 0.23$, $\mu = m_u/m_s \approx 0.62$.
b) These are the values used in the fitting.
c) Experimental widths in brackets are calculated from the branching ratios of ref. [2], and the lifetimes (in 10^{-13} s) $\tau_{D^+} = 10.3^{+10.5}_{-4.1}$, $\tau_{D^0} = 1.0^{+0.43}_{-0.27}$ of ref. [3].

ly phenomenological study, we will allow g_s and g_o to vary independently. In the following we first discuss the hadronic two-body decays and then discuss the inclusive decays.

The two-body decay amplitude can be written as

$$A = a + bg_s + cg_o, \qquad (6)$$

where a, b, c are completely known, and g_s and g_o are determined by fitting the data. Expressions for different two-body decay amplitudes of D and F mesons are given in column 2 of table 1. The decay widths are given by:

$$\Gamma = \frac{(GM_D^2 F_\pi)^2}{32\pi M_D} (f^c \chi_+ \cos^2\theta_c)^2 |A|^2$$

$$\approx 2.0 (f^c \chi_+)^2 |A|^2 \times 10^{11}\ \mathrm{s}^{-1}, \qquad (7)$$

where light meson masses have been neglected. We have $M_D = 1.86$ GeV, the pion decay constant $F_\pi = 0.96\, m_\pi$, equal K and η decay constants $f = F_K/F_\pi = F_\eta/F_\pi = 1.23$, $\tan\theta_c = 0.23$ and have ignored mass differences in the phase-space calculation. From QCD [11] it is estimated that $c_- \approx 2.1$ and $c_+ = 1/\sqrt{c_-} \approx 0.70$. However, analyses of strange particle decays indicate that there is a strong renormalization which further suppresses the symmetric operator [12]. Consequently we make the simple assumption that $c_+ = 0.70/2 \approx 0.35$.

Using the decay widths of $D^0 \to K^-\pi^+$, $D^0 \to \bar{K}^0\pi^0$ and $D^+ \to K^0\pi^+$ deduced from emulsion experiments [3] and SPEAR data [2], we have determined the following parameters:

$$f^c = 0.93, \quad g_s = -0.90, \quad g_o = -2.40, \qquad (8)$$

which are then used to predict widths of other decay modes. Although the decay constant f^c is not known precisely, the leptonic partial widths [1,2] of the D mesons suggest that it should be of the order of unity. The parameters g_s and g_o measure the color-singlet and -octet transition strengths in the decay processes.

The detailed expressions given in table 1 show that for the Cabibbo-favored D^0 decays the octet contributions are further helped by the renormalization factor $y_1 = f_+/\chi_+ \approx 1.31$ while the singlet terms are reduced by the factor $x = \chi_-/\chi_+ \approx -0.50$. The measured ratio of the rates (in brackets) of the two fastest decay modes, $D^0 \to K^-\pi^+$ and $D^0 \to \bar{K}^0\pi^0$, requires that both g_s and g_o be negative and $|g_s| < |g_o|$. Just as it is designed to do, the model predicts this ratio to have the proper magnitude and leaves the width of $D^+ \to \bar{K}^0\pi^+$ appropriately small. For the Cabibbo-suppressed decays, the widths for $D^0 \to K^-K^+$, $\pi^-\pi^+$ and $D^+ \to \bar{K}^0K^+$ are in agreement with data but the ratio $\Gamma(D^0 \to K^-K^+)/\Gamma(D^0 \to \pi^-\pi^+)$ appears to be slightly too small. This ratio should be sensitive to final state effects not included here [8].

The widths for $F^+ \to \bar{K}^0K^+$ and $\eta\pi^+$ are predicted to be somewhat smaller than the $D^0 \to K\pi$ widths, but much greater than the $D^+ \to K\pi$ width. From the measured lifetime of F^+ [3] we expect both branches to be at the 4% level. Note that in the SU(3) limit, $\Gamma(F^+ \to \eta\pi^+)/\Gamma(D^+ \to \bar{K}^0K^+) = \frac{2}{3}\cot^2\theta_c \approx 12$. In our calculation SU(3) is broken in such a way that this ratio is enhanced by a factor of 2 to 3. Numerically, we have found the widths of $F^+ \to \bar{K}^0K^+$, $\eta\pi^+$ and $D^+ \to \bar{K}^0K^+$ to be most sensitive to, and to increase rapidly with g_s, but the aforementioned ratio remains stable. Since the width for $\Gamma(D^+ \to \bar{K}^0K^+)$ is known experimentally, in our model it is difficult for $\Gamma(F^+ \to \bar{K}^0K^+, \eta\pi^+)$ not to be of the order of 1 to 2×10^{11} s^{-1}.

We now compute the total and inclusive decay rates. The total decay width is

$$\Gamma = (G^2 m_c^5/192\pi^3)(\gamma_L + \gamma_{sp} + \gamma_{ns}), \qquad (9)$$

where

$$\gamma_L = 2, \quad \gamma_{sp} = 2c_+^2 + c_-^2,$$

$$\gamma_{ns} = \cos^2\theta_c (\cos^2\theta_c\ \gamma_A + \sin^2\theta_c\ \gamma_S)$$

are respectively the semi-leptonic, spectator and non-spectator contributions. In units of $3g_0^2\chi_+^2K$ the non-spectator reduced widths are given by: for D^0,

$$\gamma_A = [(1+\mu)xg]^2 + y_1^2,$$

$$\gamma_S = (2xg)^2(1+\mu^2) + (2y_1)^2;$$

for D^+,

$$\gamma_A = 0, \quad \gamma_S = (2g)^2 + y_2^2;$$

and for F^+,

$$\gamma_A = (2g)^2 + (\mu y_2)^2, \quad \gamma_S = [(1+\mu)g]^2 + (\mu y_2)^2;$$

$g = g_s/g_o$, and the rest of the notation is the same as in table 1. The factor K arises from the difference in the kinematics of sp and ns processes; numerically $K \approx 1$. Using again the values given in (8), the results in

Table 2
Results for total and inclusive decay rates.

	Lifetime (10^{-13} s) (m_c = 1.5 GeV)	Inclusive branching ratios (%)		
		Leptonic	Hadronic	
			allowed	suppressed
D^0	$1.2 \begin{pmatrix} 1.01^{+0.43}_{-0.27}\ ^{a)} \\ <2.1\ ^{b)} \end{pmatrix}$	$3.0 \begin{pmatrix} <4\ ^{b)} \\ 5.2 \pm 3.3\ ^{c)} \end{pmatrix}$	$84\ (76 \pm 14\ ^{c)})$	$9.5\ (>7.9 \pm 2.6\ ^{c)})$
D^+	$5.2 \begin{pmatrix} 10.3^{+10.5}_{-4.1}\ ^{a)} \\ 10.4^{+4.9}_{-2.9}\ ^{b)} \end{pmatrix}$	$13 \begin{pmatrix} 22^{+4.4}_{-2.2}\ ^{b)} \\ 15.8 \pm 5.3\ ^{c)} \end{pmatrix}$	$54\ (61 \pm 19\ ^{c)})$	$20\ (> 5.6 \pm 2.9\ ^{c)})$
F^+	$2.1\ (2.2^{+2.8}_{-1.0}\ ^{a)})$	5.0	84	5.2

[a] Ref. [3]. [b] Ref. [1]. [c] Ref. [2].

table 2 are obtained, with available data given in brackets. The quantitative agreement between theory and data is certainly encouraging because, as can be seen from (7) and (9), the exclusive and inclusive widths are calculated quite differently. The experimental branching ratios for the Cabibbo-suppressed decays are lower limits since they were determined from only the K^+ contents in D decays [2]. Our calculated F^+ lifetime of 2.6×10^{-13} s is in good agreement with the only piece of data [3] on F^+ decays. The pattern of the calculated inclusive branching ratios for F^+ is distinct from those of the D mesons. In particular it has an intermediate leptonic branch ($\approx 10\%$) but has the smallest Cabibbo-suppressed hadronic branch ($\approx 5\%$).

In summary, although it is certain that the decay mechanism proposed here does not give the whole picture of a very complex situation, we have shown that it can quite simply and plausibly reproduce all the inclusive and non-leptonic two-body decay data of the charmed mesons. The model also makes definite predictions about F^+ decays which can be tested by experiment.

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