Nuclear Physics B207 (1982) 301–312

THE QCD LEADING BEHAVIOUR OF THE POLARIZED PHOTON STRUCTURE FUNCTIONS AND POSSIBLE EXPERIMENTAL TEST

H.C. LEE

Atomic Energy of Canada Ltd., Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada

LI Bing-An, SHEN Qi-Xing and ZHANG Mei-Man

Institute of High Energy Physics, Academia Sinica, China

YU Hong[1]

International School for Advanced Studies (SISSA), Trieste, Italy

Received 8 December 1981
(Revised 14 June 1982)

In this paper we calculate the leading contributions to the polarized photon structure functions using general Altarelli–Parisi equations. We also discuss the possible experimental test for asymmetry in the two-photon production of hadrons via the process $e^+ + e^- \to e^+ + e^- + x$.

1. Introduction

During the last few years, discussion about the photon structure functions has caused a great deal of interest [1]. The reason is partly theoretical, but mostly experimental. The photon structure functions are accessible experimentally in the process $e^+ + e^- \to e^+ + e^- +$ hadrons. Moreover, they can be calculated directly from QCD.

The unpolarized photon structure functions are studied by Dewitt et al. [2] using general Altarelli–Parisi equations.

In this paper we display results for the polarized photon structure functions in the three-, four-, and five-flavour cases and point out the possibility of experimental test in the two-photon production of hadrons via polarized electron beam collision.

2. The polarized photon structure functions

As the energy of e^+e^- storage rings is increased, two-photon production of hadrons via the process $e^+e^- \to e^+e^-x$ has caused much interest. The main reason is that the two-photon process cross section rises logarithmically, while the one-photon annihilation cross section falls as $1/S$ (S is the square of the total c.m.

[1] Permanent address: Institute of High Energy Physics, Academia Sinica, China.

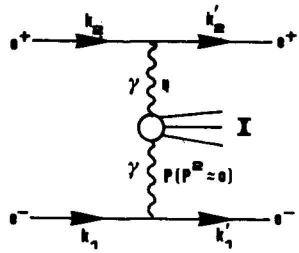

Fig. 1. Feynman diagram for the space-like $e^+ + e^- + e^+ + e^- + x$ process. One of the virtual photons is nearly on-mass-shell.

energy of the e^+e^- system). Therefore, even though the former is a process of fourth order in the fine structure constant and the latter is of second order in α_γ, as S reaches higher values (for example at PETRA energy) the cross section of the former is largely beyond the latter.

We know that the space-like process has a most important contribution to the cross section for S sufficiently large and one photon chosen nearly on-mass-shell (see fig. 1). This process includes the (virtual) $\gamma_1^* + \gamma_2^* \to x$ transition and the $1 \to 1' + \gamma_1^*$, $2 \to 2' + \gamma_2^*$ vertices. The differential cross section can be written as

$$d\sigma = \frac{e^4}{p^4 q^4} \tau_{\rho\tau}^{(1)} \tau_{\nu\mu}^{(2)} W^{\rho\tau\nu\mu} \frac{1}{16E_1'E_2'(k_1 \cdot k_2)} \frac{d^3k_1'\, d^3k_2'}{(2\pi)^6}, \tag{1}$$

where [5]

$$\begin{aligned} W^{\rho\tau\nu\mu} &= 2P_0 \int d^4x\, e^{iqx} \langle \gamma_{\lambda'} | j_q^\mu(x) j_q^\nu(0) | \gamma_\lambda \rangle \epsilon_\lambda^\rho(p) \epsilon_{\lambda'}^{\tau *}(p) \\ &= \tfrac{1}{2}(W_{++,++} + W_{+-,+-}) R^{\rho\tau} R^{\nu\mu} + \tfrac{1}{2}(W_{++,++} - W_{+-,+-})(R^{\rho\nu}R^{\tau\mu} - R^{\tau\nu}R^{\rho\mu}) \\ &\quad + \tfrac{1}{2}W_{++,--}(R^{\rho\nu}R^{\tau\mu} + R^{\rho\mu}R^{\tau\nu} - R^{\rho\tau}R^{\nu\mu}) + W_{+0,+0}R^{\rho\tau}Q_2^\nu Q_2^\mu \\ &= 4\pi \left\{ W_1 R^{\rho\tau} R^{\nu\mu} + \left(\frac{(p \cdot q)^2}{-q^2} W_2 - W_1 \right) R^{\rho\tau} Q_2^\nu Q_2^\mu \right. \\ &\quad \left. + \tfrac{1}{2}W_3(R^{\rho\nu}R^{\tau\mu} + R^{\rho\mu}R^{\tau\nu} - R^{\rho\tau}R^{\nu\mu}) + W_4(R^{\rho\nu}R^{\tau\mu} - R^{\tau\nu}R^{\rho\mu}) \right\}. \end{aligned} \tag{2}$$

$W_{a'b',ab}$ is called the helicity amplitude:

$$\begin{aligned} R^{\nu\mu} &= -g^{\nu\mu} + \frac{1}{X}\{q \cdot p(p^\mu q^\nu + q^\mu p^\nu) - q^2 p^\mu p^\nu\}, \\ X &= (p \cdot q)^2, \\ Q_2 &= \sqrt{\frac{-q^2}{X}} \left(p - q \frac{p \cdot q}{q^2} \right). \end{aligned} \tag{3}$$

If we consider the deep inelastic scattering process of polarized electrons on polarized photons, taking the Bjorken limit $Q^2=-q^2\to\infty$, $\nu=p\cdot q\to\infty$ with $x=Q^2/2\nu$ fixed and by using the naive parton model we have

$$\begin{aligned} W_1 &= \sum_{i=1}^{2f} \tfrac{1}{2}e_i^2 q^i(x,t)\,,\\ \nu W_2 &= \sum_{i=1}^{2f} e_i^2 x q^i(x,t)\,,\\ W_3 &= 0\\ W_4 &= -\tfrac{1}{2}\sum_{i=1}^{2f} e_i^2[q_+^i(x,t)-q_-^i(x,t)]\,, \end{aligned} \tag{4}$$

where e_i is the charge of ith quark in units of the electron charge. $q^i(x,t)=q_+^i(x,t)+q_-^i(x,t)$, $q_{r_i}^i(x,t)$ is the density of ith quark with helicity r_i in a photon of positive polarization. f is the number of the quark flavours. If we take the average over the target photon polarization, the W_4 term is cancelled. So we know that W_4 is a structure function connected directly with the target photon polarization. The W_3 term is relevant to the interference of two transverse polarizations of the target photon. W_3 would be zero in the naive parton model.

3. The leading behaviour of the photon structure functions at large Q^2

Dewitt et al. [2] discussed the leading contributions to the unpolarization structure functions of a real or slightly virtual photon using a generalization of the Altarelli–Parisi equations [3] for QCD scale breaking. We discovered that the colour factor "3", which is neglected by them, is important because it appears in the anomalous term that is found to be dominant for the leading behaviour of the photon structure functions.

Recently Berger et al. [4] presented results on deep inelastic electron-photon scattering at momentum transfers $1<Q^2<15$ GeV, using data taken at PETRA. The results are expressed in terms of the photon structure function F_2 and are compared with QCD predictions. The data are clearly consistent with the QCD prediction (of course the colour factor can't be neglected) in leading order.

Thus it can be seen that the discussion of the polarized photon structure function is very interesting. It can give us more information about photon structure.

Omitting the higher order in the electromagnetic interactions, then we have the following Altarelli–Parisi type equations

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\Delta q^i(x,t) &= 3e_i^2\frac{\alpha_\gamma}{2\pi}\Delta P_{q\gamma}(\lambda)+\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{\mathrm{d}y}{y}\left[\Delta q^i(y,t)\Delta P_{qq}\left(\frac{x}{y}\right)+\Delta G(y,t)\Delta P_{qG}\left(\frac{x}{y}\right)\right],\\ \frac{\mathrm{d}}{\mathrm{d}t}\Delta G(x,t) &= 0+\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{\mathrm{d}y}{y}\left[\sum_{ji=1}^{2f}\Delta q^i(y,t)\Delta P_{Gq}\left(\frac{x}{y}\right)+\Delta G(y,t)\Delta P_{GG}\left(\frac{x}{y}\right)\right], \end{aligned} \tag{5}$$

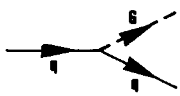

Fig. 2. The quark–gluon vertex which determines ΔP_{Gq} and ΔP_{qq}.

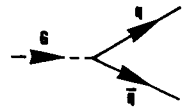

Fig. 3. The annihilation vertex of a gluon into a quark–antiquark pair which determines ΔP_{qG}.

where $3e_i^2(\alpha_\gamma/2\pi)\Delta P_{q\gamma}(x)$ is an anomalous term and this is a point-like contribution arising from the fact that photons can create "free" $\mathrm{q\bar{q}}$ pairs. The factor 3 comes from the colour, $\Delta P_{\mathrm{q}\gamma}(x)=P_{\mathrm{q}_+\gamma_+}(x)-P_{\mathrm{q}_-\gamma_+}(x)$. Here we fixed the photon with positive helicity. These terms in eqs. (5) come from figs. 2, 3, 4, and 5 respectively. As we omit the higher order in the electromagnetic interactions, the contribution of fig. 6 can be neglected.

Simply we have the relation

$$\Delta P_{\mathrm{q}\gamma}(x)=2\Delta P_{\mathrm{qG}}(x)\,, \tag{6}$$

while [3]

$$\begin{aligned}
\Delta q^i &= q^i_+ - q^i_-\,, \qquad \Delta G = G_+ - G_-\,,\\
\Delta P_{AB} &= P_{A_+B_+} - P_{A_-B_+}\,,\\
\Delta P_{\mathrm{qq}}(z) &= \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+}+\tfrac{3}{2}\delta(z-1)\right],\\
\Delta P_{\mathrm{Gq}}(z) &= \frac{4}{3}\,\frac{1-(1-z)^2}{z}\,,\\
\Delta P_{\mathrm{qG}}(z) &= \tfrac{1}{2}[z^2-(1-z)^2]\,,\\
\Delta P_{\mathrm{GG}}(z) &= 3\left[(1+z^4)\left(\frac{1}{z}+\frac{1}{(1-z)_+}\right)-\frac{(1-z)^3}{z}+(\tfrac{11}{6}-\tfrac{1}{9}f)\delta(z-1)\right].
\end{aligned} \tag{7}$$

The solution of eqs. (5) in the asymptotic limit is

$$\begin{aligned}
\Delta q^i(x,t) &= \frac{\alpha_\gamma}{2\pi}a^i(x)t\left[1+\mathrm{O}\left(\frac{1}{t}\right)\right],\\
\Delta G(x,t) &= \frac{\alpha_\gamma}{2\pi}b(x)t\left[1+\mathrm{O}\left(\frac{1}{t}\right)\right].
\end{aligned} \tag{8}$$

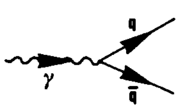

Fig. 4. The annihilation vertex of a photon into a quark–antiquark pair which determines $\Delta P_{\mathrm{q}\gamma}$.

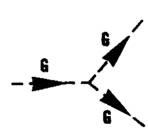

Fig. 5. The three-gluon vertex to determine ΔP_{GG}.

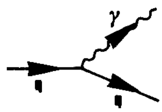

Fig. 6. The quark electromagnetic vertex.

Substituting into eqs. (5), we find the integral equations

$$a^i(x)=3e_i^2\Delta P_{q\gamma}(x)+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\left[\Delta P_{qq}\left(\frac{x}{y}\right)a^i(y)+\Delta P_{qG}\left(\frac{x}{y}\right)b(y)\right],$$
$$b(x)=0+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\left[\Delta P_{Gq}\left(\frac{x}{y}\right)\sum_{j=1}^{2f}a^j(y)+\Delta P_{GG}\left(\frac{x}{y}\right)b(y)\right]. \tag{9}$$

Notice that the running coupling constant $\alpha_s(t)=\alpha_s(0)/(1+\alpha_s(0)bt)\underset{t\to\infty}{\to}1/bt$, where $b=(11-\frac{2}{3}f)/4\pi$, $t=\ln(Q^2/\Lambda^2)$.

These equations are t-independent. For simplifying the calculation and considering that quarks (or antiquarks) with equal squared electric charges have equal $a^i(x)$ from eqs. (5), we may define

$$a^{NS}(x)=\tfrac{1}{2}(a^u(x)-a^d(x)),$$
$$a^S(x)=\frac{1}{2f}\sum_{i=1}^{2f}a^i(x). \tag{10}$$

The eqs. (5) become

$$a^{NS}(x)=\tfrac{1}{2}\Delta P_{q\gamma}(x)+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\Delta P_{qq}\left(\frac{x}{y}\right)a^{NS}(y),$$
$$a^S(x)=D_f\Delta P_{q\gamma}(x)+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\left[\Delta P_{qq}\left(\frac{x}{y}\right)a^S(y)+\Delta P_{qG}\left(\frac{x}{y}\right)b(y)\right], \tag{11}$$
$$b(x)=0+\frac{1}{2\pi b}\int_x^1\frac{dy}{y}\left[2f\Delta P_{Gq}\left(\frac{x}{y}\right)a^S(y)+\Delta P_{GG}\left(\frac{x}{y}\right)b(y)\right],$$

where $D_f=(3/2f)\sum_{i=1}^{2f}e_i^2=\frac{11}{15},\frac{5}{6},\frac{2}{3}$ for $f=5,4,3$ respectively.

For the different f we have the following relations:

$$f=5:\quad a^u(x)=a^s(x)+\tfrac{6}{5}a^{NS}(x),$$
$$a^d(x)=a^S(x)-\tfrac{4}{5}a^{NS}(x);$$
$$f=4:\quad a^u(x)=a^S(x)+a^{NS}(x),$$
$$a^d(x)=a^S(x)-a^{NS}(x); \tag{12}$$
$$f=3:\quad a^u(x)=a^S(x)+\tfrac{4}{3}a^{NS}(x),$$
$$a^d(x)=a^S(x)-\tfrac{2}{3}a^{NS}(x).$$

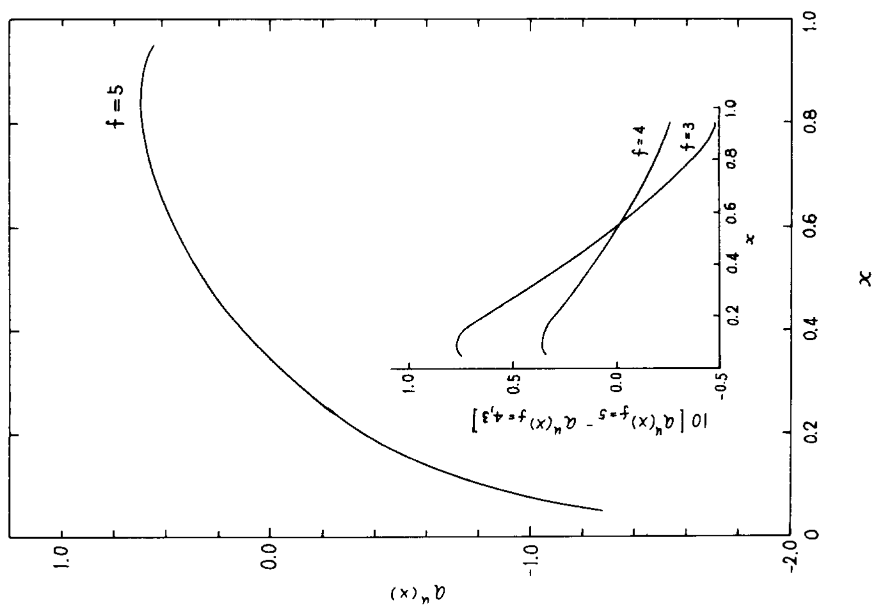

Fig. 7. The u-quark function $a^u(x)$ for flavour groups SU(f), $f = 5$, 4, 3. See eqs. (11) and (12).

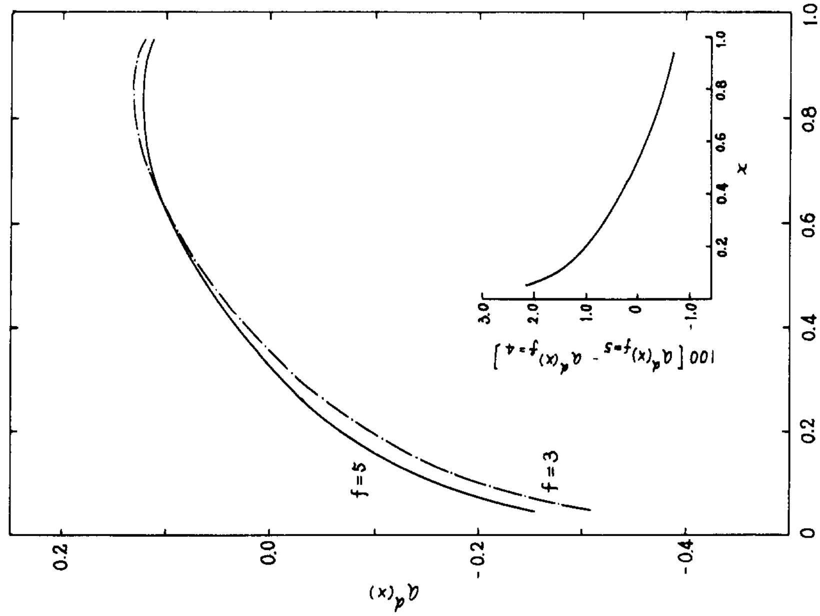

Fig. 8. The d-quark function $a^d(x)$, see eqs. (11) and (12).

Solving eqs. (11) numerically and using eqs. (12), we may get the leading terms $a^{u}(x)$, $a^{d}(x)$, $b(x)$ of $\Delta q^{i}(x,t)$ and $\Delta G(x,t)$. The results are shown in figs. 7, 8 and 9 respectively. We have also

$$W_4 = -\tfrac{1}{2}\sum_{i=1}^{2f} e_i^2 \Delta q^i(x,t) = -\frac{\alpha_\gamma}{4\pi} A(x) t\left[1 + \mathrm{O}\left(\frac{1}{t}\right)\right]. \tag{13}$$

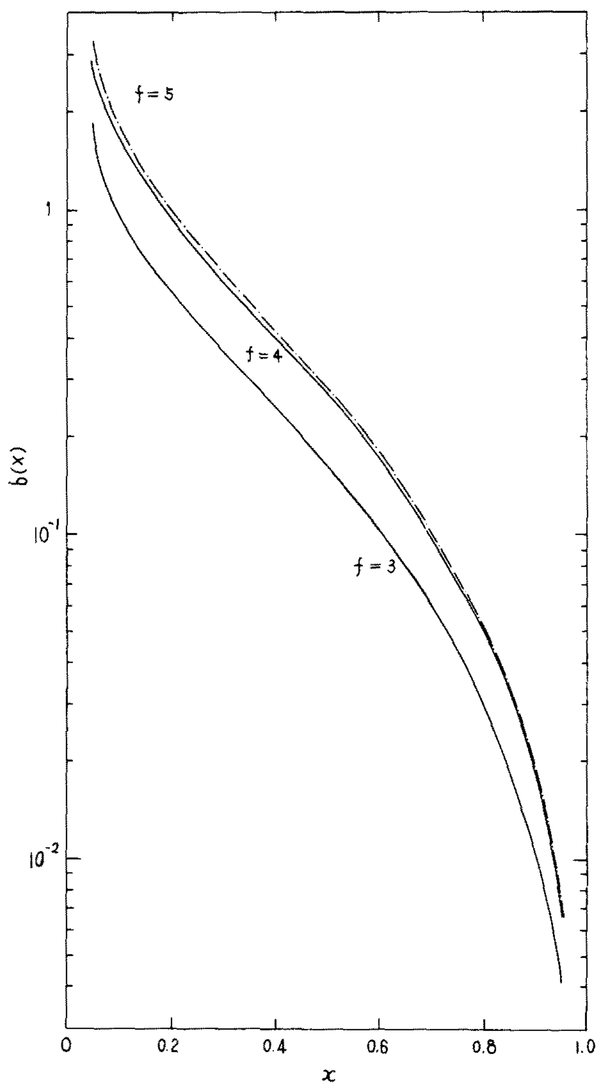

Fig. 9. The gluon function $b(x)$, see eqs. (11) and (12).

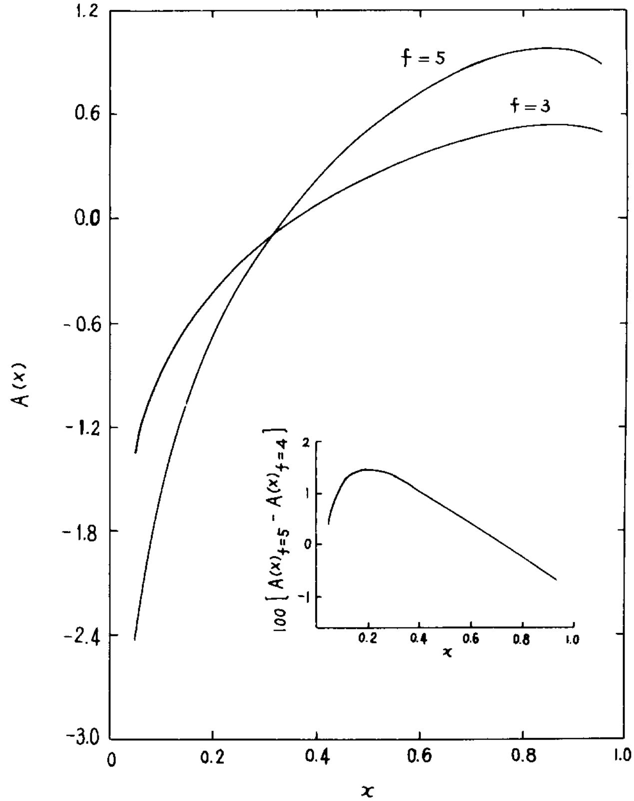

Fig. 10. The leading term $A(x)$ of the polarized photon structure function W_4, see eqs. (13) and (14).

Its leading terms (see fig. 10) are

$$A(x)=\sum_{i=1}^{2f} e_i^2 a^i(x)=\begin{cases}\frac{22}{9}a^{\mathrm{S}}(x)+\frac{8}{5}a^{\mathrm{NS}}(x)\,, & f=5\,,\\ \frac{20}{9}a^{\mathrm{S}}(x)+\frac{4}{3}a^{\mathrm{NS}}(x)\,, & f=4\,,\\ \frac{4}{3}a^{\mathrm{S}}(x)+\frac{8}{9}a^{\mathrm{NS}}(x)\,, & f=3\,.\end{cases} \tag{14}$$

We may also take the moment of each side of eqs. (11). The moment equations obtained may be solved algebraically:

$$\begin{aligned} a_n^{\mathrm{NS}} &= \tfrac{1}{2}\Delta P_n^{\mathrm{q}\gamma}+\frac{1}{2\pi b}\Delta P_n^{\mathrm{qq}}a_n^{\mathrm{NS}}\,,\\ a_n^{\mathrm{S}} &= D_f\Delta P_n^{\mathrm{q}\gamma}+\frac{1}{2\pi b}(\Delta P_n^{\mathrm{qq}}a_n^{\mathrm{S}}+\Delta P_n^{\mathrm{qG}}b_n)\,,\\ b_n &= \frac{1}{2\pi b}(2f\Delta P_n^{\mathrm{Gq}}a_n^{\mathrm{S}}+\Delta P_n^{\mathrm{GG}}b_n)\,, \end{aligned} \tag{15}$$

where

$$a_n^{\mathrm{NS}} = \int_0^1 x^{n-1} a^{\mathrm{NS}}(x)\,\mathrm{d}x\,,$$

$$a_n^{\mathrm{S}} = \int_0^1 x^{n-1} a^{\mathrm{S}}(x)\,\mathrm{d}x\,,$$

$$b_n = \int_0^1 x^{n-1} b(x)\,\mathrm{d}x\,,$$

$$\Delta P_n^{\mathrm{q}\gamma} = 2\Delta P_n^{\mathrm{qG}} = 2\int_0^1 \mathrm{d}z z^{n-1}\Delta P_{\mathrm{qG}}(z) = \frac{n-1}{n(n+1)}\,, \tag{16}$$

$$\Delta P_n^{\mathrm{qq}} = \int_0^1 \mathrm{d}z z^{n-1}\Delta P_{\mathrm{qq}}(z) = \frac{4}{3}\left[-\frac{1}{2}+\frac{1}{n(n+1)}-2\sum_{j=2}^{n}\frac{1}{j}\right],$$

$$\Delta P_n^{\mathrm{Gq}} = \int_0^1 \mathrm{d}z z^{n-1}\Delta P_{\mathrm{Gq}}(z) = \frac{4}{3}\frac{n+2}{n(n+1)}\,,$$

$$\Delta P_n^{\mathrm{GG}} = \int_0^1 \mathrm{d}z z^{n-1}\Delta P_{\mathrm{GG}}(z) = 3\left[\frac{11}{6}-\frac{f}{9}+\frac{2}{n}-\frac{4}{n+1}-2\sum_{j=1}^{n-1}\frac{1}{j}\right].$$

The solutions are

$$a_n^{\mathrm{NS}} = \tfrac{1}{2}\Delta P_n^{\mathrm{q}\gamma}\Big/\left(1-\frac{1}{2\pi b}\Delta P_n^{\mathrm{qq}}\right),$$

$$a_n^{\mathrm{S}} = D_f \Delta P_n^{\mathrm{q}\gamma}\left(1-\frac{1}{2\pi b}\Delta P_n^{\mathrm{GG}}\right)\Big/\left[1-\frac{1}{2\pi b}(\Delta P_n^{\mathrm{GG}}+\Delta P_n^{\mathrm{qq}}) + \frac{1}{(2\pi b)^2}(\Delta P_n^{\mathrm{qq}}\Delta P_n^{\mathrm{GG}} - 2f\Delta P_n^{\mathrm{qG}}\Delta P_n^{\mathrm{Gq}})\right],$$

$$b_n = \frac{f}{\pi b}\Delta P_n^{\mathrm{Gq}} a_n^{\mathrm{S}}\Big/\left(1-\frac{1}{2\pi b}\Delta P_n^{\mathrm{GG}}\right). \tag{17}$$

The moment of W_4 is

$$\langle W_4\rangle_n = -\frac{\alpha_\gamma}{4\pi}A_n t\left[1+\mathrm{O}\left(\frac{1}{t}\right)\right],$$
$$A_n = \sum_{i=1}^{2f} e_i^2 a_n^i\,. \tag{18}$$

4. A possible experimental test

In order to distinguish this process from background, we must therefore detect at least one of the scattered electrons and at the same time detect some of the produced hadrons. If we take the initial electrons at very small angles and define

$$y_1=\frac{p\cdot q}{k_1\cdot q},\qquad y_2=\frac{q\cdot p}{k_2\cdot p},\qquad \varepsilon_1=\frac{1-y_1}{1-y_1+\frac{1}{2}y_1^2},\qquad \varepsilon_2=\frac{1-y_2}{1-y_2+\frac{1}{2}y_2^2},$$

omitting the electron mass, the differential cross section has the form

$$\mathrm{d}\sigma=\frac{(4\pi\alpha)^2}{p^2q^2}\,\frac{\xi}{32k_1k_2E_1'E_2'}\,\frac{\mathrm{d}^3k_1'\,\mathrm{d}^3k_2'}{(2\pi)^6},\tag{19}$$

where

$$\xi=\frac{64\pi}{y_1^2y_2^2}(1-y_1+\tfrac{1}{2}y_1^2)(1-y_2+\tfrac{1}{2}y_2^2)\left\{2W_1+2\varepsilon_2\left(\frac{\nu W_2}{2x}-W_1\right)+\varepsilon_1\varepsilon_2\cos 2\phi W_3\right\}$$
$$+32\lambda_1\lambda_2\frac{\pi}{y_1y_2}(4-2y_1-2y_2+y_1y_2)W_4\,.\tag{20}$$

ϕ is the azimuthal angle between the scattering electrons in the colliding electron's c.m.s, $\lambda_j=(-1)^{\frac{1}{2}+r_j}$, r_j is the helicity of the jth electron, $k_j=|\boldsymbol{k}_j|$. Obviously the W_4 term is relevant to the polarized electron beams collision. When we integrate the azimuthal angle, the W_3 term disappears.

Define the asymmetry

$$\hat{A}=\frac{W_4}{W_1}.\tag{21}$$

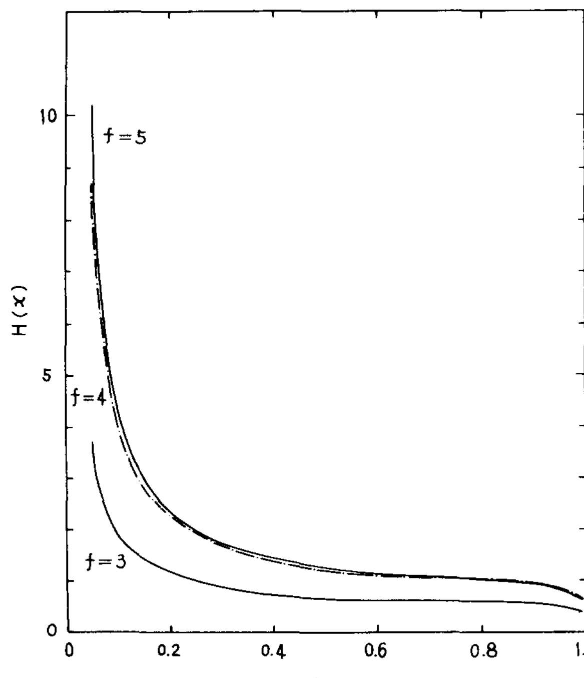

Fig. 11. The leading term of the photon structure function W_1.

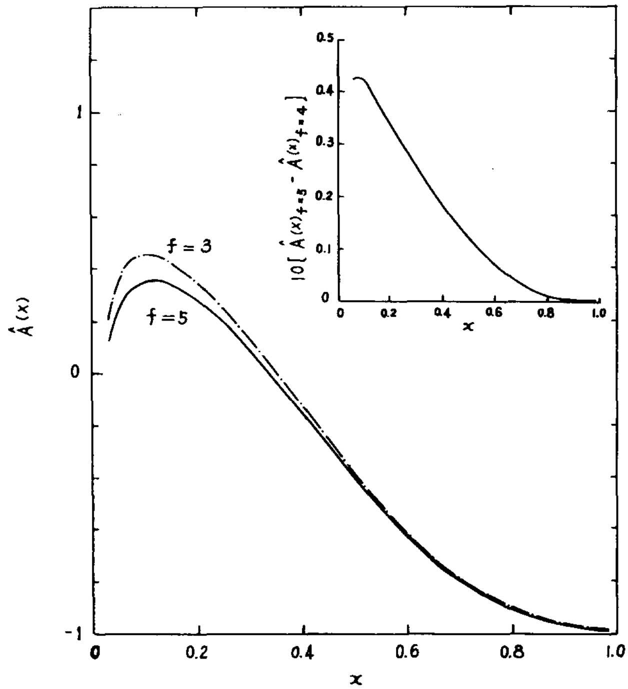

Fig. 12. The asymmetry. See eq. (22).

When Q^2 is large enough. We have

$$\hat{A} = -\frac{A(x)}{H(x)}, \tag{22}$$

where $H(x)$ is the leading term of the photon structure function $W_1 = (\alpha_\gamma/4\pi)H(x)t[1+O(1/t)]$. Its behaviour is shown in fig. 11. $\hat{A}$ is a quantity which can be tested by experiment (see fig. 12).

The strange behaviours of the W_4 leading term (from positive to negative as x increases) and the Bjorken scales, apart from an overall logarithmic factor for the photon structure functions, are different from the proton structure functions. The proton structure functions contract to $x = 0$ at $Q^2 \to \infty$. The strange behaviour is entirely due to the effect of the anomalous component, because we know that as Q^2 is large enough the contribution of the anomalous component dominates.

One of the authors (Y.H.) would like to thank Professor P. Budinich and Professor R. Iengo for hospitality at the International School for Advanced Studies, Trieste where this work was completed.

References

[1] S.J. Brodsky et al., Phys. Rev. Lett. 27 (1971) 280; Phys. Rev. D19 (1979) 1418;
T.F. Walsh, Phys. Lett. 36B (1971) 121; 44B (1973) 195;
C. Peterson et al., Nucl. Phys. B174 (1980) 424;
E. Witten, Nucl. Phys. B120 (1977) 189;
C.H. Llewellyn-Smith, Phys. Lett. 79B (1978) 83;
J. Gilman, SLAC-PUB 2597 (1980);
C.-M. Wu, CERN preprint (1979)
[2] R.J. Dewitt et al., Phys. Rev. D19 (1979) 2046
[3] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298
[4] Ch. Berger et al., DESY 81-051
[5] V.M. Budnev et al., Phys. Reports 15 (1975) 181