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QUENCHING OF AXIAL-VECTOR COUPLING CONSTANT IN THE β-DECAY OF FINITE NUCLEI

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Abstract: The meson-exchange-current contributions to the Gamow-Teller matrix element in closed-shell-plus- (or minus) one nuclei are calculated and expressed in terms of effective coupling constants multiplying standard rank-one spherical tensors. The dependence of the quenching of g_A on the orbit and mass of the nucleus is determined.

1. Introduction

Recent analyses [1]) of the Gamow-Teller (GT) transitions in light nuclei suggest that the axial-vector coupling constant, g_A, may be quenched in these nuclei. The quenching of g_A is believed to arise from two sources: (a) core-polarization and configuration-mixing effects and (b) from the meson-exchange-current effects [2,3]). The configuration effects can be minimised by studying only nuclei with one particle or one hole outside a closed shell ($N = Z$) core. For these same nuclei, first order core-polarization effects are zero. Higher order effects may be expected to be small except for the results of Shimizu *et al.* [4]) who claim that the tensor component of the two-body force can give sizeable contributions in second-order of perturbation theory. However it has been conjectured by Rho [2]) that these tensor force contributions may be cancelled by those second-order terms with a nucleon and a Δ-particle in the intermediate state. This conjecture of Rho still needs to be proved.

In this note we calculate the quenching of g_A in finite nuclei due to the meson exchange currents. It should be emphasized that the change of the GT matrix elements with nuclear mass A due to the meson-exchange currents has been calculated previously [5]) and the magnitudes have been found to vary violently with A. It has been suggested [3]) that the quenching of g_A due to the meson-exchange current is uncertain in magnitude and sign implying that the natural connection of the Lorentz-Lorenz force [2]) in π-nucleus scattering and the quenching of g_A in nuclear matter is lost in the case of finite nuclei where the surface-dependent effects play an important role. The present work is an attempt to re-express these GT matrix elements in such a manner that the quenching of g_A due to explicit surface-dependent effects can be displayed. Then it is possible to establish a connection between the quenching of g_A and the Lorentz-Lorenz force in π-scattering from finite nuclei.

The PCAC (partially conserved axial-vector current) hypothesis has been used to relate [2] the quenching of g_A in nuclear matter to the Lorentz-Lorenz force in π-nucleus scattering. In the case of nuclear matter, i.e. constant matter density, the quenching is estimated [2] to be $\approx 22\,\%$ implying that $g_A^{\text{eff}} = 0.96$ as compared to a value of $g_A = 1.23$ for the decay of a free neutron. In fact the quenching is a non-linear function of the nuclear-matter density, ρ, and in magnitude is given as $(1+c\rho)^{-1}$. The constant c is related to g', the constant that gives the spin-isospin component of the quasi-particle interaction in Migdal's theory [6] of finite nuclei, which has been obtained by fitting experimental data on magnetic moments and M1 transitions. Rho [2] has used an estimate of $\frac{1}{3}$ for g' while the phenomenological estimates of Migdal [6] give a value of 0.5. More realistic estimates [7] suggest a value of 0.4–0.5 for g'. Choosing a value for g' of 0.5, the estimated quenching of g_A in nuclear matter will be $\approx 35\,\%$. With this background it appears of paramount interest to study the quenching of g_A in finite nuclei by explicitly including the surface-dependent effects. The point is that the valence nucleons, which actually undergo β-decay; have a large probability of occupying a region where the matter density is not constant but is a rapidly varying function of the radial distance r.

In order to calculate the change (δg_A) in the axial-vector coupling constant in finite nuclei, we choose several odd-A nuclei with a single-valence particle or a hole outside LS closed shells. The contribution to the GT matrix element of the valence particle or hole from the exchange currents is calculated by summing over the core. Then the three matrix elements between the single-particle states $j = l\pm\frac{1}{2}$, spin-orbit partners, are re-expressed in terms of all tensors of rank-1 with positive parity; i.e. the effective GT transition operator is written [8] as

$$\mp\tfrac{1}{2}[\delta g_A \boldsymbol{\sigma} + \delta g_l \boldsymbol{l} + \sqrt{8\pi}\Gamma_P (Y_2\times\boldsymbol{\sigma})^1]\tau_\pm, \tag{1}$$

where the upper sign refers to β^- decay and the lower sign to β^+ decay. The isospin raising and lowering operators are defined as $\tau_\pm = \tau_x \pm i\tau_y$. This effective operator is to be compared with the definition of the one-body GT transition operator $\mp\frac{1}{2}g_A\boldsymbol{\sigma}\tau_\pm$. The three constants δg_A, δg_l and Γ_P are determined from the three matrix elements between the single-particle state $j = l\pm\frac{1}{2}$. Then δg_A directly gives a measure of the change in g_A.

In sect. 2 the expression for the two-body GT operator as derived by Chemtob and Rho [9] is given. The various constants are defined for completeness. In sect. 3 the results of our calculation are presented along with the conclusions that can be drawn with respect to the quenching of g_A.

2. Theory

The reduced matrix element of the λ-pole two-body operator, $\mathcal{O}^{(2)}_\lambda$, between the single-particle states j_1 and j_1' is given as (with isospin factors understood):

$$\langle j_1'\|\mathcal{O}^{(2)}_\lambda\|j_1\rangle = \sum_{j_c J_1 J_1'} \frac{\hat{J}_1\hat{J}_1'}{\hat{j}_1\hat{j}_1'} U(j_1' j_c \lambda J_1; J_1' j_1)\langle j_1' j_c; J_1'\|\mathcal{O}^{(2)}_\lambda\|j_1 j_c; J_1\rangle, \tag{2}$$

where $\hat{J} = \sqrt{2J+1}$, and the reduced matrix elements are defined according to the conventions of Brink and Satchler [10]. The two-body meson-exchange operator for GT transitions ($\lambda = 1$) is given [9] by

$$\begin{aligned}\mathcal{O}^{(2)}_{\lambda} = -\tfrac{1}{2}g_{\mathrm{A}}\{(\tau(1)\times\tau(2))_{\pm}[(\sigma_1\times\sigma_2)g_{\mathrm{I}}+T^{(\times)}_{12}g_{\mathrm{II}}]\\ +[\tau(1)-\tau(2)]_{\pm}[(\sigma_1-\sigma_2)(h_{\mathrm{I}}+h^{\sigma}_{\mathrm{I}}P^{\sigma}_{12})+T^{(-)}_{12}(h_{\mathrm{II}}+h^{\sigma}_{\mathrm{II}}P^{\sigma}_{12})]\\ +[\tau(1)+\tau(2)]_{\pm}[(\sigma_1+\sigma_2)j_{\mathrm{I}}+T^{(+)}_{12}j_{\mathrm{II}}+\Sigma_{12}j_{\mathrm{III}}]+H_{\mathrm{NL}}\},\end{aligned} \tag{3}$$

where

$$\Sigma_{12} = \tfrac{1}{3}i[(\sigma_1\cdot\hat{r})(\sigma_2\times\hat{r})+(\sigma_1\times\hat{r})(\sigma_2\cdot\hat{r})],$$

$$T_{12} = [(\sigma_1\odot\sigma_2)\cdot\hat{r}\hat{r}-\tfrac{1}{3}\sigma_1\odot\sigma_2], \qquad \odot = \pm,\ \times,$$

$$P^{\sigma}_{12} = \tfrac{1}{2}(1+\sigma_1\cdot\sigma_2).$$

Here H_{NL} is the non-local part of the two-body exchange operator that is ignored in the present calculation. The upper (lower) sign refers to β^- (β^+) decay.

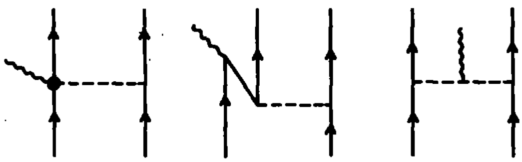
The form for the radial functions for the non-Born term and the pair-exchange term are given [9] as follows:

(a) Non-Born term: One-pion exchange (fig. 1a)

$$g_{\mathrm{I}} = \tfrac{2}{3}\xi\alpha(0)Y_0(x_\pi), \qquad g_{\mathrm{II}} = -\xi\alpha(0)Y_2(x_\pi),$$

$$h_{\mathrm{I}} = j_{\mathrm{I}} = -\tfrac{1}{6}\xi\gamma(0)Y_0(x_\pi), \qquad h_{\mathrm{II}} = j_{\mathrm{II}} = -\tfrac{1}{2}\xi\gamma(0)Y_2(x_\pi),$$

$$h^{\sigma}_{\mathrm{I}} = h^{\sigma}_{\mathrm{II}} = j_{\mathrm{III}} = 0,$$



(a) (b) (c)

Fig. 1. Some of the diagrams for the meson-exchange-current contribution to the weak decay (solid line = nucleons, dotted line = mesons and wiggly lines = weak current): (a) non-Born; (b) pair-exchange, and (c) mesonic exchange.

where

$$\xi = \frac{1}{8\pi}\frac{g_r(0)}{g_{\mathrm{A}}}\frac{m_\pi^3}{M}, \qquad Y_0(x) = \frac{e^{-x}}{x},$$

$$Y_2(x) = \left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y_0(x), \qquad x_\pi = m_\pi r,$$

with $g_r\,(q^2 = 0)$ being the pion-nucleon coupling constant off-the-energy-shell where q^2 is the four-momentum transferred. The on-shell pion-nucleon coupling constant, $g_r(q^2 = -m_\pi^2)$, has a magnitude of 13.6 and $g_A = 1.23$. Here m_π and M are the mass of the pion and nucleon respectively. We use $\hbar = c = 1$.

The PCAC implies that the constants $\alpha(0)$, $\beta(0)$ and $\gamma(0)$ are related to the non-Born π-nucleon scattering amplitudes. The magnitude of these constants have been estimated by Adler [11]) by extrapolating the on-shell π-nucleon scattering amplitude and are given as (in units of m_π^2)

$$\alpha(0) = 0.72, \qquad \beta(0) = -0.79, \qquad \gamma(0) = 3.2. \tag{4}$$

These constants have also been estimated by postulating a phenomenological Lagrangian [9]) to describe the effective coupling for the vertices πNN and ANN, where A is the axial-vector current. In this model the magnitude of the constants are

$$\alpha(0) = 1.158, \qquad \beta(0) = -1.021, \qquad \gamma(0) = 2.519. \tag{5}$$

It may be interesting to recall that the constants $\alpha(0)$ and $\beta(0)$ are related by Adler-Weisberger [12]) relation

$$-\frac{2M^2}{g_r(0)g_A}[\alpha(0)+\beta(0)] = 1-\frac{1}{g_A^2}. \tag{6}$$

(b) Pair-excitation term: One-pion exchange [9]) (fig. 1b),

$$g_I = 2h_I^\sigma = \frac{2}{3}\frac{m_\pi}{M} f_{\pi NN}^2 Y_0(x_\pi),$$

$$g_{II} = 2h_{II}^\sigma = -\tfrac{2}{3}j_{III} = -\frac{m_\pi}{M} f_{\pi NN}^2 Y_2(x_\pi),$$

$$h_I = h_{II} = j_I = j_{II} = 0.$$

where

$$f_{\pi NN}^2 = \frac{1}{4\pi}\left(g_r\,\frac{m_\pi}{2M}\right)^2 \approx 0.08.$$

It should be remarked that

(i) One-pion exchange current (fig. 1c) does not contribute due to conservation of G-parity;

(ii) Contributions due to recoil (fig. 2a) and wave-function renormalisation graphs (fig. 2b) are neglected. It has been shown by Gari and Hyuga [13]) that these contributions cancel each other to the degree of approximation used here;

(iii) Heavy meson-exchange graphs obtained by replacing the pion line by the heavy mesons (ρ, ω, . . ., etc.) are also neglected because these will lead to operators that are much shorter ranged and hence are damped out by the presence of short-range

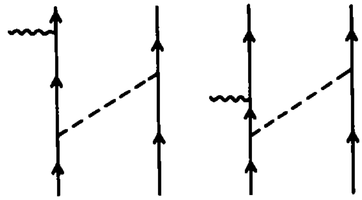


Fig. 2. Examples of Feynman diagrams, (a) for wave-function renormalisation and (b) for recoil corrections.

correlations in the nuclear wave functions arising from the strongly repulsive core in the two-body nuclear force.

3. Results and summary

Antisymmetrised two-body matrix elements of the exchange-current operator $\mathcal{O}^{(2)}_{\pm}$ have been calculated in a harmonic-oscillator basis with $\hbar\omega = 41A^{-\frac{1}{3}}$ MeV. In practice we found it easier to calculate the two-body matrix elements in the LS coupling scheme. The short-range correlations are simply dealt with by cutting off the integral at $r = 0.4$ fm. In the cases of orbitals for which the two-body matrix elements are convergent, such a cut-off introduces a very small change in the value of the matrix element. We have chosen to look at the decay rate of one particle or one hole $N = Z$ closed-shell nuclei with $A = 4$, 16, 40, 80, 140 and 224. The last three nuclei are not realised in nature. In addition we look at the decay of a nucleon in one of the inner orbits. This way we can learn about the variation of g_A with l of the single-particle orbit and with A of the nucleus.

The results of the analysis of the one-body matrix elements are shown in table 1. Several valence-particle orbitals and all orbitals with one hole in the LS closed core are considered. Several comments should be made about the results:

(a) The pair term is much smaller ($\approx$ 5–10 % of the total contribution shown in table 1) than the non-Born term in all cases considered. In the non-Born terms, the results obtained with constants $\alpha(0)$ and $\gamma(0)$ derived from the PCAC prescription are larger in magnitude than those obtained with constants derived from the phenomenological Lagrangian by about a factor of two (table 1).

(b) In all cases $\delta g_l \equiv 0$. This arises from the fact that the two-body exchange operator (neglecting H_{NL}) is Galilean invariant. It should be recalled that in the case of the exchange-current contributions to the M1 operator the one-pion exchange contribution (fig. 1c) is not zero and the resultant two-body operator is Galilean noninvariant. This is the cause for the non-zero contribution of fig. 1c to δg_l in the case of M1 transitions.

TABLE 1

Variation of δg_A and Γ_P with the single-particle orbit and with A [a])

	$A = 4$		$A = 16$		$A = 40$		$A = 80$		$A = 140$		$A = 224$	
	δg_A	Γ_P	δg_A	Γ_P	δg_A	Γ_P	δg_A	Γ_P	δg_A	Γ_P	δg_A	Γ_P
0s	0.007		−0.068		−0.143		−0.206		−0.256		−0.294	
	0.046		−0.001		−0.054		−0.100		−0.136		−0.165	
0p	−0.044	0.181	−0.071	0.116	−0.124	0.074	−0.176	0.049	−0.222	0.034	−0.260	0.025
	−0.023	0.114	−0.023	0.068	−0.053	0.039	−0.088	0.023	−0.119	0.013	−0.146	0.008
0d			−0.080	0.139	−0.112	0.096	−0.154	0.066	−0.194	0.047	−0.229	0.035
			−0.047	0.085	−0.058	0.054	−0.081	0.033	−0.106	0.020	−0.130	0.012
1s			−0.101		−0.129		−0.165		−0.202		−0.235	
			−0.066		−0.072		−0.091		−0.113		−0.135	
0f					−0.106	0.110	−0.137	0.080	−0.171	0.058	−0.203	0.043
					−0.065	0.065	−0.079	0.042	−0.097	0.026	−0.117	0.016
1p					−0.131	0.105	−0.156	0.080	−0.185	0.060	−0.213	0.045
					−0.087	0.057	−0.095	0.040	−0.109	0.026	−0.125	0.017
0g							−0.123	0.090	−0.151	0.068	−0.180	0.052
							−0.077	0.050	−0.091	0.033	−0.107	0.021
1d							−0.150	0.088	−0.172	0.069	−0.195	0.054
							−0.100	0.045	−0.108	0.032	−0.119	0.021
2s							−0.161		−0.180		−0.202	
							−0.109		−0.115		−0.125	
0h									−0.134	0.075	−0.160	0.058
									−0.086	0.039	−0.098	0.026
1f									−0.161	0.079	−0.180	0.060
									−0.107	0.036	−0.116	0.025

First line: PCAC; second line: phenomenological Lagrangian.

[a]) The PCAC results quoted in ref. [8]) have an opposite sign (due to the definition of the effective operator) and are smaller in magnitude (due to a smaller value for $\hbar\omega$ and for the pion-Compton wavelength) than the results quoted here.

(c) The terms h_1^{σ}, h_{11}^{σ} and j_{111} in the operator $\mathcal{O}_{\lambda}^{(2)}$, eq. (3), are non-hermitian and give zero contribution in the present calculation.

(d) For s-orbitals $\Gamma_P \equiv 0$. In the case of the lowest s-orbital, δg_A increases in magnitude as A is increased and may saturate for heavy nuclei ($A > > 224$). It appears that for s-orbitals g_A is quenched by $\gtrsim 25\,\%$ for very large nuclei. (For realistic closed-shell nuclei with $N \neq Z$, such as ^{208}Pb, there are orbitals like $1h_{\frac{9}{2}}$ for protons and $1i_{11/2}$ for neutrons that are unfilled while their spin-orbit partners are filled. These have the problem of inducing first-order core-polarization effects that are large. For the cases that we have considered core-polarization effects are zero in first order of perturbation theory.)

(e) For non s-state orbitals both δg_A and Γ_P are non-zero. In all cases while δg_A increases in magnitude, Γ_P decreases in magnitude as A is increased. In fig. 3, we show the variation of δg_A and Γ_P for the 0s and 0p states as the mass number of the $N = Z$ closed-shell nucleus is varied.

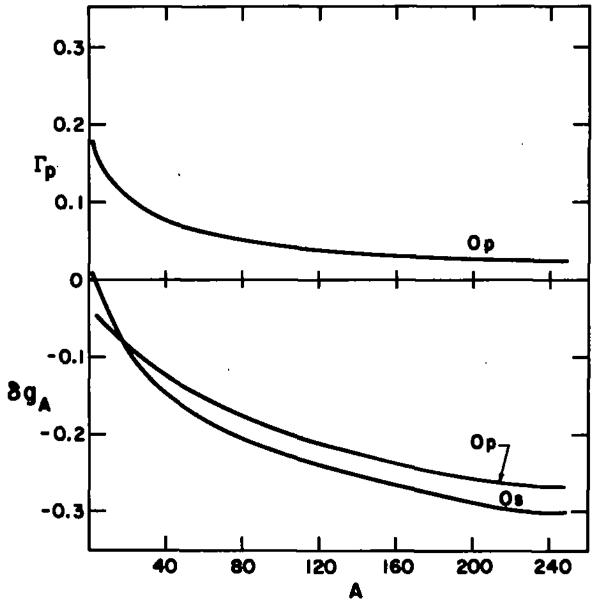


Fig. 3. Variation of Γ_p for the lowest p-state and variation of δg_A for the lowest s- and p-states with mass number A.

(f) Even though the magnitude of the one-body matrix elements for the various cases studied do not show any definite trend with the mass number or with the orbital of the single particle or the single hole, a parametrisation of the results in terms of the effective GT operator given in eq. (1) leads to a smooth variation of δg_A and Γ_p with mass number and with the orbital of the single-particle state.

(g) For the valence-particle orbitals δg_A is smaller than for the deep-lying states. We find that g_A is quenched by about 15 % for the valence orbitals for $A > 16$. This suggests that quenching of g_A depends on the density of the nuclear system. It is the averaging of the two-body operator $\mathcal{O}^{(2)}_\lambda$ by the probability distribution of the single particle in its orbit and with the density of the remaining core nucleus that leads to this variation in δg_A. Certainly the density of the core nucleus is nearly constant for an inner-lying single-particle orbital; this is not so for an orbital of a valence nucleon. The magnitude of quenching of g_A for a β-decaying nucleon in an inner-orbit can be compared closely to the quenching of g_A calculated for nuclear matter. For example, fig. 3 suggests that a reduction of ≥ 25 % in g_A for a 0s orbital in heavy nuclei, a little larger than the ≈ 22 % estimate of Rho [2]) but less than the later estimates of ≈ 35 % using Migdal's theory. Of course, the nuclear density in a finite nucleus near the origin is only on the average the same as that for nuclear matter. There is as well a fluctuating component implying regions exist where the density is greater than that of nuclear matter. This may be the reason that δg_A does not appear to saturate for a very heavy nucleus.

In summary the meson-exchange-current contributions expressed by an effective one-body operator [eq. (1)] indicate that the axial-vector coupling constant is quenched to a varying extent depending on the orbit, mass of the nucleus and the prescription for calculating the non-Born contributions.

The surface-dependent effects are taken into account explicitly and the magnitude of δg_A given in table 1 indicates the quenching of g_A in finite nuclei. The results give a natural extension of the results of Rho [2]) that the quenching of g_A in nuclear matter is related to the Lorentz-Lorenz force in the π-nuclear matter scattering. The quenching of g_A is a smooth function of A and of the density of nuclei in the region of the decaying nucleon.

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