

THE NUCLEAR NEUTRON-PROTON rms RADIUS DIFFERENCE

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Abstract: The Hartree-Fock method is used to calculate the n-p rms radius differences for eight nuclei from ^{48}Ca to ^{120}Sn . These values are compared with experiment and with those predicted from Coulomb displacement energies. The comparison is discussed.

Recently there has been some controversy over the differences between the rms radii of neutron and proton distributions in neutron-rich nuclei. The analysis of Auerbach, Quershi and Steinheim ¹⁾ of the 700 MeV $\pi^\pm$ -Pb inelastic scattering data showed that $r_n = (0.980 \pm 0.015)r_p$; an attempt to fit the data assuming r_n to be more than 5 % larger than r_p leads to a very peculiar and unreasonable neutron distribution. The conclusion of Auerbach *et al.* contradicted the results obtained by Greenlees *et al.* ²⁾ in their first optical-model analysis of the proton elastic scattering data, and also those obtained by Elton ³⁾ from a fitting of the bound-state energy levels in the single-particle Woods-Saxon model ⁴⁾ for Pb; both these analyses gave $r_n \approx 1.09r_p$. It was also in disagreement with the work of Nolen, Schiffer and Williams ⁵⁾, who deduced that $r_n - r_p \approx 0.07 \pm 0.03$ fm for Pb by assuming (i) that the distribution of the extra proton in the analogue state is the same as the distribution of the excess neutrons in the parent state, (ii) that the Coulomb displacement energy due to this proton is equal to the energy difference between the analogue and parent states, and (iii) that isobaric spin is a good quantum number.

More recently Greenlees *et al.* ⁶⁾ recalculated the p-elastic scattering data and concluded that the n-p rms radius differences are much smaller than those they had previously deduced, the new results being essentially zero for nuclei with $A \approx 60$, and ≈ 0.15 fm for ^{120}Sn and ^{208}Pb . This implies $r_n \approx 1.02 r_p$ for Pb.

Now the radii of the proton and neutron distributions reflect the dynamical equilibrium of the nucleons under the influence of the Coulomb and nucleon-nucleon interactions, and it is desirable to study this equilibrium in a self-consistent way. We have therefore carried out Hartree-Fock (HF) calculations for the nuclei ^{48}Ca , ^{56}Fe , ^{58}Ni , ^{59}Co , ^{60}Ni , ^{62}Ni , ^{90}Zr and ^{120}Sn . For the N-N interaction in the Hamiltonian we use the semi-realistic potential no. 2 of Saunier and Pearson ⁷⁾. This potential has a phenomenological short-range part and a one-pion exchange long-range part. It

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TABLE 1
Hartree-Fock binding energies and rms charge radii

Nucleus	Binding energy (MeV)		$\langle r_{\text{ch}}^2 \rangle^{\frac{1}{2}}$ (fm)		
	Hartree-Fock	Experiment ^{a)}	Hartree-Fock	e^- scatt. ^{b)}	μ^- X-rays
⁴⁸ Ca	365.15	416.00	3.284	3.49	
⁵⁶ Fe	443.05	492.26	3.633	$3.67 \approx 3.85$	3.72 ± 0.03 ^{c)} $3.663 \approx 3.764$ ^{d)}
⁵⁸ Ni	456.34	506.46	3.695	$3.72 \approx 3.93$	3.81 ± 0.03 ^{c)}
⁵⁹ Co	465.73	517.32	3.671	$3.76 \approx 4.05$	3.81 ± 0.06 ^{c)}
⁶⁰ Ni	476.24	526.85	3.702	$3.75 \approx 3.84$	3.85 ± 0.03 ^{c)}
⁶² Ni	493.23	545.27	3.703		3.88 ± 0.03 ^{c)}
⁹⁰ Zr ^{e)}	696.44	799.74	3.951		4.27 ± 0.04 ^{c)}
¹²⁰ Sn	911.03	1020.55	4.653	$4.57 \approx 4.65$	4.65 ± 0.01 ^{c)} $4.593 \approx 4.672$ ^{d)}

^{a)} Ref. ²³⁾.

^{b)} Ref. ²⁴⁾.

^{c)} Ref. ²⁶⁾, calculated from $r_0 = A^{\frac{1}{3}}/R$ listed for natural elements.

^{d)} Ref. ²⁵⁾.

^{e)} Values calculated with $\hbar\omega = 10.5$ MeV are shown. With $\hbar\omega = 8.0$ MeV, $E_b = 677.74$ MeV, $\langle r_{\text{ch}}^2 \rangle^{\frac{1}{2}} = 4.410$ fm. We believe the optimum value for $\hbar\omega$ to be between 10.5 and 8.0 MeV.

TABLE 2
Difference of rms radii of neutron and proton density distributions, $\langle r_n^2 \rangle^{\frac{1}{2}} - \langle r_p^2 \rangle^{\frac{1}{2}}$ in fm

Nucleus	p-elastic scattering ^{a)}	Hartree-Fock this work	others	Droplet model ^{e)}	Statistical theory ^{f)}	Coulomb displacement energy	Proton Woods-Saxon potential ¹⁾	Neutron separation energy
⁴⁸ Ca		0.137	0.24 ^{b)} 0.23 ^{c)}	0.166	0.10	0.06 ¹⁾	0.25	
⁵⁶ Fe	-0.04 ± 0.15	0.074		0.063				0.24 ^{k)}
⁵⁸ Ni	0.01 ± 0.18	0.020		0.023				0.14 ^{k)}
⁵⁹ Co	-0.03 ± 0.16	0.088		0.078				
⁶⁰ Ni	0.03 ± 0.16	0.065		0.058				0.23 ^{k)}
⁶² Ni		0.114		0.092		0.04 ¹⁾	0.12	
⁹⁰ Zr		0.150	0.13 ^{b)} 0.12 ^{c)}	0.111		0.02 ± 0.01 ^{h)}		
¹²⁰ Sn	0.15 ± 0.19	0.246		0.202		0.05 ± 0.02 ^{h)} 0.07 ¹⁾	0.25	0.44 ^{k)}
²⁰⁸ Pb [†]	0.13 ± 0.25		0.29 ^{b)} 0.23 ^{c)} 0.25 ^{d)}	0.254	0.11	0.07 ± 0.03 ^{g)} 0.09 ± 0.01 ^{h)} 0.15 ¹⁾ [*]	0.28	0.65 ± 0.5 ^{j)} 0.62 ^{k)}

^{a)} Ref. ⁶⁾.

^{b)} Ref. ¹⁰⁾.

^{c)} Ref. ¹¹⁾.

^{d)} Ref. ¹²⁾.

^{e)} Ref. ¹⁴⁾.

^{f)} Ref. ¹⁵⁾.

^{g)} Ref. ⁵⁾.

^{h)} Ref. ¹⁶⁾.

ⁱ⁾ Ref. ¹⁷⁾.

^{j)} Ref. ³⁾.

^{k)} Ref. ¹⁸⁾.

^{*} Also, Bethe and Siemen ²⁰⁾ found that the half-density radius of neutrons is 0.2 fm less than that of the protons in ²⁰⁸Pb.

[†] The value -0.11 ± 0.08 was obtained in ref. ¹⁾.

accounts for certain aspects of the N-N scattering data and gives correct binding and saturation results in nuclear matter calculations⁷⁾. In the HF Hamiltonian the kinetic energy due to the c.m. motion is subtracted explicitly and the Coulomb interaction is included exactly. The harmonic-oscillator basis for the expansion of the single-particle wave functions includes those in the first five major shells as well as the $1h_{1/2}$ shell. This basis can in principle accommodate 82 protons and 82 neutrons. The single-particle orbitals are expanded in terms of harmonic-oscillator wave functions with no restrictions except for definite parity and for the prohibition of mixing of proton and neutron orbitals. The value of $\hbar\omega$ is 13.5 MeV for ^{48}Ca , 10.5 for the $A \approx 60$ nuclei, and 8.0 for ^{120}Sn . Calculations carried out for ^{90}Zr with 10.5 and with 8.0 MeV lead to similar results. The trial wave functions required to start the HF iterative procedure are generated from the single-particle deformed oscillator code ELMOS written by Cusson⁹⁾. The calculated binding energies and *charge* radii, relative to the c.m. are listed in table 1 and compared with experimental data. The binding energies are too small²³⁾ by $\approx 10\%$ and the rms charge radii too small²⁴⁻²⁶⁾ by 3-6%. Since we are only concerned with the n-p rms radius difference within each nucleus we feel these discrepancies should not invalidate our results for this difference.

Table 2 compares various estimates of the n-p radius difference for nuclei from ^{48}Ca to ^{208}Pb . Those deduced from p-elastic scattering by Greenlees *et al.*⁶⁾ appear in the first column. The second and third columns give the present and other HF results¹⁰⁻¹²⁾. The results in column 4 are those predicted from the refined liquid-drop ("droplet") model of Myers and Swiatecki^{13,14)}. The values calculated from the statistical theory by Brueckner *et al.*¹⁵⁾ are given in column 5 while column 6 lists results deduced from the Coulomb displacement energies of analogue states^{5,16,17)}, using the method of Nolen *et al.*⁵⁾. In column 7 we find the results of Nolen and Schiffer, calculated by using Woods-Saxon potentials characterized by the rms radius of the relevant proton core¹⁷⁾. Finally, column 8 gives results obtained from single-particle potentials determined by the separation energies of the relevant neutrons [refs. ^{3,18)}]. The numbers in column 8 are obviously much larger than those obtained by any other method and will be excluded from the following discussion[†].

The p-elastic scattering results are in agreement with all those in columns 2-7, but cannot distinguish between them. We can group the HF, droplet model, and statistical calculation into one category because they all involve some form of the variational principle. There is excellent agreement among the HF results themselves and between the HF and droplet-model results, but they are larger than the results of the statistical theory. The proton densities in the latter calculation tend to be much more prone to expansion under the influence of the Coulomb force. For example the proton rms radius is predicted to be 0.05 fm larger than the neutron rms radius for ^{40}Ca , as compared to 0.03 fm in our calculation and 0.01 fm in the droplet model. This seems to imply a smaller compressibility coefficient in the statistical model of Brueckner

[†] The method itself is also subject to criticism, see ref. ²⁰⁾.

et al. The reason for the discrepancy between these models is not well understood. Henceforth we shall be content to compare the HF and droplet-model results with those obtained by the two remaining methods.

There is good agreement between the variational results and those of column 7. This is somewhat surprising because, unlike the Woods-Saxon type calculations, the variational calculations do not assume the proton and neutron "core" distributions to be identical. Yet when we calculate the rms radii of neutron "cores" we find that these are only 0.02 to 0.03 fm or 0.6 % less than the proton radii. This may explain the similarity between the Woods-Saxon and HF results.

Comparison of columns 2-4 with column 6 reveals a systematically small but significant difference. The numbers in the latter are 2-3 times smaller than those in the former. This discrepancy between the neutron-excess rms radii extracted from Coulomb displacement energies and those predicted by variational calculations was also observed by Nolen and Schiffer¹⁷). The most serious criticism of a HF calculation for medium and heavy nuclei is that the single-particle basis may be too small, thereby restricting the proton distribution from expanding beyond the distribution of the neutrons in corresponding orbitals. Short of doing a proper calculation by enlarging the HF basis and as a result greatly increasing the size of the computer program, one can try to estimate the increase in r_p which could occur if the basis restrictions were removed. For nucleides with A up to 40, the effect of the basis restriction is minimal. For these nuclei, our HF calculations show, to a very good approximation[†], that

$$r_p - r_n = 4 \times 10^{-3} A^{\frac{1}{3}} r_p \quad (N=Z). \quad (1)$$

This difference is, incidentally, about three times larger than the prediction of the droplet model in this mass region. If we extrapolate eq. (1) to heavier nuclei, and consider the r_n in eq. (1) to be the rms radius of the *core* neutrons, then for Fe, Ni, Zr, Sn, and Pb the rms radius differences between the protons and *core* neutrons would be 0.04, 0.04, 0.06, 0.07 and 0.09 fm, respectively. Since as mentioned earlier the rms radii of the protons are already ≈ 0.02 fm larger than those of the neutron cores, the corrections to r_p , or $\langle r_{ch} \rangle^{\frac{1}{3}}$, should be 0.02, 0.02, 0.04, 0.05, and 0.07 fm, respectively, for these nuclei. Subtracting these corrections from the corresponding numbers in column 2 of table 2 we see that in most cases this only brings our HF results somewhat closer to those of the droplet model but does not eliminate the discrepancies between the variational results and the Coulomb energy results.

Although there are indeed other corrections which should be applied to the HF predictions we should mention that there are also difficulties in the Coulomb energy method. For example, an important step of that method is to identify the differences in binding energies of analogue states with the differences in expectation values of the Coulomb interaction. Whereas this equality is predicted by Nolen and Schiffer¹⁷)

[†] We are not concerned with the absolute optimum value for $\hbar\omega$ here. A more elaborate report of a systematic HF calculation for light nuclei will be published elsewhere.

to be within several keV we have found [†] that the binding-energy difference is larger by ≈ 90 keV for $A = 41$. Also, the structural relations between analogue states of nuclei with large neutron excesses are more complicated and rearrangement energy corrections may well be significantly larger. Indeed, if we take the values (consistent with experiment) of the n-p rms radius differences for ^{90}Zr , ^{120}Sn and ^{208}Pb to be 0.1, 0.2 and 0.25 fm, respectively, then, we find that, from the $\Delta E_{\text{Coulomb}}$ versus $\langle r_{\text{exp}} \rangle^{\frac{1}{2}}$ curves of Friedman and Mendelbaum ¹⁶⁾, the analogue energy differences that cannot be accounted for by Coulomb energies alone are ≈ 0.9 , ≈ 0.6 and ≈ 0.4 MeV, respectively. These represent 8 %, 4 % and 2 % of the Coulomb displacement energies, or 0.11 %, 0.06 %, and 0.02 % of the total binding energies of the respective nuclei. That the energy discrepancies should decrease for heavier nuclei may seem peculiar at first but may be in accord with the proposition that the isospin impurity of a nucleidic state decreases as the neutron excess increases ^{21, 22)}.

To summarize the present discrepancy could be due to some unexpected fault of the HF method. On the other hand, the first-order Coulomb method may be at fault. N. Auerbach *et al.* ¹⁹⁾ recently studied some corrections terms in first-order perturbation theory, and found that the net correction *reduced* $\Delta E_{\text{Coulomb}}$. However, there are corrections that cannot be reliably estimated in perturbation theory. An example is the rearrangement energy corrections mentioned above. Moreover, recently Damgaard *et al.* ⁸⁾ studied the correction to $\Delta E_{\text{Coulomb}}$ from monopole polarization due to the isovector part of the Coulomb interaction. They found the correction to be $100 \approx 700$ keV. For ^{208}Pb the correction is 600 keV, more than enough to erase the discrepancy mentioned above. Further detailed studies of these and other possible corrections should be made before definite conclusions can be drawn about the neutron-proton rms radius difference.

[†] See preceding footnote.

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