

The Analytic, Principal-Value and Landshoff Prescriptions for Axial Gauges

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The word prescription is itself suggestive: it is a device used to deal with something that, under ideal circumstances, should not be there. In the case of axial gauges [1], the intruders are spurious singularities induced in Feynman integrals by a noncovariant gauge choice. To see how this comes about, consider the pure Yang-Mills Lagrangian with a gauge fixing term $1/(2\alpha) (n \cdot A)^2$, where n is an arbitrary 4-vector with $n^2 \neq 0$,

$$\mathcal{L} = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (n_\mu A^\mu)^2 \right), \quad (1)$$

for which the propagator, defined as the reciprocal of the coefficient of the term quadratic in the gauge field A , is in the $\alpha \rightarrow 0$ limit

$$D_{\mu\nu}^{(0)}(p) = \frac{1}{p^2} \left[-g_{\mu\nu} + \frac{p_\mu n_\nu + p_\nu n_\mu}{p \cdot n} - \frac{n^2 p_\mu p_\nu}{(p \cdot n)^2} \right], \quad (2)$$

where indices of the gauge group have been suppressed. (We are only interested in the $\alpha \rightarrow 0$ limit. In our view, the axial gauge loses much of its merit - such as the decoupling of the Faddeev-Popov ghost - when $\alpha \neq 0$.) This propagator has two types of singularities: (i) at $p^2=0$, and (ii) at $p \cdot n=0$. The first type is physical and is the origin of infrared divergence in the amplitudes of certain processes. The second type is induced by the noncovariant gauge fixing and is the source of spurious singularities. We say spurious because they are singularities that should not manifest itself in a physical amplitude. A physical amplitude is supposed to be gauge independent, and had we chosen a covariant gauge to work in, the Feynman integrals for such an amplitude would not have such singularities. The reason that we do encounter such singularities when calculating physical amplitudes in axial gauges

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has something to do in the way we do calculations, at least in perturbation theory: An amplitude that is not singular is decomposed as a sum of Feynman integrals, some of which may be singular. The singularities ought to cancel among themselves when the Feynman integrals are combined to give the amplitude. A well-known example occurs in the computation of the γ_5 -anomaly (or any other anomaly) in perturbation theory. The anomaly itself is finite, but typically it is decomposed by partial fraction into two easier-to-evaluate but singular integrals. (Taylor [2] has shown us an elegant way to extract the γ_5 -anomaly without hard computation.) The function of a prescription is to isolate the spurious gauge-fixing singularity from the physical infrared singularity. This task is made considerably more complicated by the entanglement of the $p^2=0$ and $p \cdot n=0$ poles at the point $p=0$.

The complication arises because the $p^2=0$ singularity also needs to be regularized by a prescription. One commonly used is the Feynman prescription in which p^2 is replaced by $p^2+i\epsilon'$. This would ensure that a particle propagates forward in time. Furthermore, the ultraviolet divergence at $p \rightarrow \infty$ also has to be regularized. All told there are three categories of singularities that need to be separately regularized:

- (i) The gauge induced spurious singularities must all cancel at the end of a calculation for any physical amplitude.
- (ii) The $p^2=0$ singularities are physical; their noncancellation at the end of the calculation for an amplitude is an indication that that amplitude is infrared divergent. (Here we shall not differentiate between infrared divergence and a zero-mass singularity.)
- (iii) The ultraviolet divergences must be collected as data for the renormalization program.

This discussion makes the point clear that not only would a prescription be incorrect if the effect of it were just to throw the infrared and ultraviolet singularities out from each individual integral. It would be equally incorrect if it simply eliminated the spurious singularities in individual integrals.

In this paper we illustrate the point by examining in some detail the analytic properties of three prescriptions for axial gauge integrals -- the principal-value prescription [3], the Landshoff [4] prescription and an analytic prescription. Landshoff's prescription is shown to be the only one that keeps separate accounts of all three categories of singularities mentioned above. The prescription is sometimes said to be ad hoc. We now argue that rather it is a natural generalization of Feynman's prescription for the $p^2=0$ pole.

First we ask how does one get Feynman's prescription if one does not want to simply replace by hand the p^2 denominator in (2) by $p^2+i\epsilon'$. It can be done as follows. One adds a small term $i\epsilon' A^2$ to the Lagrangian. In Euclidean space, this has the effect of making the functional integral over A well defined for large A^2 . Of course the added term fails to have a damping effect in Minkowski space. We shall adopt the attitude that it is sufficient for the theory to be well behaved in